Who Gets to Be In the Room? Manipulating Participation in WTO Disputes

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Supplemental Appendix

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Audience Costs

Model Structure

Continue to assume the same basic model structure from the main paper. However, assume the following payoffs:

	Early settlement	C wins panel ruling	D wins panel ruling
Complainant	$xV + n$	$V + n - k$	$-n - k$
Defendant	$(1-x)V - n$	$-n - k$	$V + n - k$

Equilibrium Behavior

Define the minimum equilibrium offer by $x_L^* \equiv x^*(\pi_L, n_H + \eta)$ and the maximum equilibrium offer by $x_H^* \equiv x^*(\pi_H, n_L)$.

For large V, there exists a fully separating equilibrium in which:

- there exists an interior cutpoint, $\tilde{\pi} \in (\pi_L, \pi_H)$, such that the complainant prevents if $\pi < \tilde{\pi}$, and promotes if $\tilde{\pi} \leq \pi$;
- equilibrium demands are $x^*(\pi, n) = \pi - \frac{2n(1-\pi)-k}{V}$; and
- the defendant settles with probability $s^*(x) = \exp\left(\frac{-(x-x_L^*)V}{2k}\right)$ for $x \in [x_L^*, x_H^*]$, $s^*(x) = 1$ for $x < x_L^*$, and $s^*(x) = 0$ for $x > x_H^*$.

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Conditional on x, the defendant will play a mixed strategy $s^*(x)$ if and only if:

$$(1-x)V - n = (1-\pi)V + n(1-2\pi) - k \quad \Leftrightarrow \quad x^*(\pi, n) = \pi - \frac{2n(1-\pi)-k}{V} \tag{1}$$

This is an interior solution for large V. Let $T_C(\pi, n)$ denote the complainant's expected utility from litigation. Then the complainant's expected utility from x is:

$$\begin{aligned} EU_C(x|\pi, n) &= s(x)(xV+n) + [1-s(x)]\, T_C(\pi, n) \\ \Rightarrow \frac{\partial EU_C(x|\pi, n)}{\partial x} &= s(x)V + s'(x)(xV+n) - s'(x)T_C(\pi, n) = 0 \\ &\Leftrightarrow \quad x = \frac{T_C(\pi, n) - n}{V} - \frac{s(x)}{s'(x)} \end{aligned} \tag{2}$$

Since both (1) and (2) must hold simultaneously in equilibrium:

$$\begin{aligned}\pi-\frac{2n(1-\pi)-k}{V} &= \frac{T_C(\pi,n)-n}{V}-\frac{s(x)}{s'(x)}\\ \Leftrightarrow \left(\frac{2k}{V}\right) s'(x) &= -s(x)\\ \Rightarrow s^*(x) &= \exp\left(\frac{-xV}{2k}+\Gamma\right)\end{aligned}$$

This is always an interior value if and only if $\Gamma \leq \frac{x_L^* V}{2k}$. Bayes' Rule does not constrain the defendant's beliefs when the complainant makes an off-the-equilibrium-path demand. We assume that the defendant always accepts very low off-the-equilibrium-path demands ($x < x_L^*$) and rejects very high off-the-equilibrium-path demands ($x > x_H^*$). No type of players ever has incentive to deviate upwards to a demand $x > x_H^*$, and no type of player ever has incentive to deviate downwards to a demand $x < x_L^*$ if and only if $\Gamma = \frac{x_L^* V}{2k}$.

The complainant's expected utility from the bargaining and litigation subgame is:

$$\begin{aligned}B_C(\pi,n) &= s^*\left(x^*(\pi,n)\right)\left[x^*(\pi,n)V+n\right]+\left[1-s^*\left(x^*(\pi,n)\right)\right]T_C(\pi,n)\\ &= \pi V-n(1-2\pi)-k+2k\exp\left(\frac{-\left[x^*\left(\pi,n\right)-x_L^*\right]V}{2k}\right)\end{aligned}$$

Note that:

$$\begin{aligned}\frac{\partial B_C(\pi,n)}{\partial n} &= 2\pi-1+2(1-\pi)\exp\left(\frac{-\left[x^*\left(\pi,n\right)-x_L^*\right]V}{2k}\right)\\ &\Rightarrow \lim_{V\to\infty}\frac{\partial B_C(\pi,n)}{\partial n}=2\pi-1>0 \quad \Leftrightarrow \quad \pi>\frac{1}{2}\end{aligned}$$

Recall that $\pi_L < \frac{1}{2} < \pi_H$. So for large V, type π_L prevents and type π_H promotes. Define:

$$\begin{aligned}\Delta(\pi) &\equiv EU_C\left(\text{promote}|\pi\right)-EU_C\left(\text{prevent}|\pi\right)\\ &= \int_{n_H}^{n_H+\eta} B_C(\pi,n)f\left(n|\text{promote}\right)dn-\int_{n_L}^{n_L+\eta} B_C(\pi,n)f\left(n|\text{prevent}\right)dn\\ &= \frac{1}{\eta}\left[\int_{n_L+\eta}^{n_H+\eta} B_C(\pi,n)dn-\int_{n_L}^{n_H} B_C(\pi,n)dn\right]\end{aligned}$$

By above, $\Delta\left(\pi_L\right)<0$ and $\Delta\left(\pi_H\right)>0$ for large V. By the implicit function theorem, there exists a cutpoint $\tilde{\pi}\in\left(\pi_L,\pi_H\right)$ such that $\Delta\left(\tilde{\pi}\right)=0$. To have an equilibrium in which all $\pi<\tilde{\pi}$ prevent and all $\tilde{\pi}\leq\pi$ promote, we must show that $\tilde{\pi}$ is unique:

$$
\begin{aligned}
\frac{\partial \Delta(\pi)}{\partial \pi} &= \frac{1}{\eta}\left[\int_{n_L+\eta}^{n_H+\eta} \frac{\partial B_C(\pi,n)}{\partial \pi} dn - \int_{n_L}^{n_H} \frac{\partial B_C(\pi,n)}{\partial \pi} dn \right] \\
&= \frac{1}{\eta}\left\{\int_{n_L+\eta}^{n_H+\eta} (V+2n)\left[1-\exp\left(\frac{-\left[x^*(\pi,n)-x_L^*\right]V}{2k}\right)\right] dn \right. \\
&\qquad \left. - \int_{n_L}^{n_H} (V+2n)\left[1-\exp\left(\frac{-\left[x^*(\pi,n)-x_L^*\right]V}{2k}\right)\right] dn \right\} \\
&\Rightarrow \lim_{V\to\infty} \frac{\partial \Delta(\pi)}{\partial \pi} > 0
\end{aligned}
$$

So for large V, $\widetilde{\pi}$ is unique.

Comparative Statics

Analogue of Proposition 1:

$$
\begin{aligned}
\frac{\partial x^*(\pi,n)}{\partial \pi} &= 1 + \frac{2n}{V} > 0 \\
\frac{\partial s^*(x)}{\partial x} &= -\exp\left(\frac{-\left(x-x_L^*\right)V}{2k}\right)\left(\frac{V}{2k}\right) < 0
\end{aligned}
$$

Analogue of Proposition 2:

$$
\frac{\partial x^*(\pi,n)}{\partial n} = -\frac{2(1-\pi)}{V} < 0
$$

Analogues of Propositions 3 and 4 follow from the equilibrium behavior described above.

Analogue of Proposition 5:

$$
\begin{aligned}
\frac{\partial s^*\left(x^*(\pi,n)\right)}{\partial n} &= \exp\left(\frac{-\left[x^*(\pi,n)-x_L^*\right]V}{2k}\right)\left(\frac{1-\pi}{k}\right) > 0 \\
\frac{\partial^2 s^*\left(x^*(\pi,n)\right)}{\partial n^2} &= \exp\left(\frac{-\left[x^*(\pi,n)-x_L^*\right]V}{2k}\right)\left(\frac{1-\pi}{k}\right)^2 > 0
\end{aligned}
$$

Analogue of Proposition 6: Recall that π is distributed according to density f on $[\pi_L, \pi_H]$. Conditional on n:

$$
\begin{aligned}
s^*\left(x^*\left(\text{promote}|n\right)\right) &= \int_{\widetilde{\pi}}^{\pi_H} s^*\left(x^*(\pi,n)\right) f\left(\pi|\text{promote}\right) d\pi \\
s^*\left(x^*\left(\text{prevent}|n\right)\right) &= \int_{\pi_L}^{\widetilde{\pi}} s^*\left(x^*(\pi,n)\right) f\left(\pi|\text{prevent}\right) d\pi
\end{aligned}
$$

The equilibrium probability of settlement is decreasing in π:

$$
\begin{aligned}
\frac{\partial s^*\left(x^*(\pi,n)\right)}{\partial \pi} &= -\exp\left(\frac{-\left[x^*(\pi,n)-x_L^*\right]V}{2k}\right)\left(\frac{V+2n}{2k}\right) < 0 \\
&\Rightarrow s^*\left(x^*\left(\text{promote}|n\right)\right) < s^*\left(x^*\left(\text{prevent}|n\right)\right)
\end{aligned}
$$

Biased Judicial Rulings

Model Structure

Continue to assume the same basic model structure and payoffs from the main paper. However, let $p(\pi, n) \in [0, 1]$ denote the probability that the complainant wins the case, where p is increasing in the complainant's type, π, and the number of third parties, n.

Equilibrium Behavior

Conditional on x, the defendant will play the mixed strategy $s^*(x)$ if and only if:

$$(1-x)V + n = [1 - p(\pi, n)]V - n[1 - 2p(\pi, n)] - k \Leftrightarrow x^*(\pi, n) = p(\pi, n) + \frac{2n[1 - p(\pi, n)] + k}{V} \tag{3}$$

This is an interior value for large V. Let $T_C(\pi, n)$ denote the complainant's expected utility from litigation. Then the complainant's expected utility from x is:

$$\begin{aligned} EU_C(x|\pi, n) &= s(x)(xV - n) + [1 - s(x)]T_C(\pi, n) \\ \Rightarrow \frac{\partial EU_C(x|\pi, n)}{\partial x} &= s(x)V + s'(x)(xV - n) - s'(x)T_C(\pi, n) = 0 \\ &\Leftrightarrow \quad x = \frac{T_C(\pi, n) + n}{V} - \frac{s(x)}{s'(x)} \end{aligned} \tag{4}$$

Since both (3) and (4) must hold simultaneously in equilibrium:

$$\begin{aligned} p(\pi, n) + \frac{2n[1 - p(\pi, n)] + k}{V} &= \frac{T_C(\pi, n) + n}{V} - \frac{s(x)}{s'(x)} \\ \Leftrightarrow \left(\frac{2k}{V}\right)s'(x) &= -s(x) \\ \Rightarrow \quad s^*(x) &= \exp\left(\frac{-xV}{2k} + \Gamma\right) \end{aligned}$$

Define $x_L^* \equiv \min\{x^*(\pi, n)\}$. The settlement probability is always an interior value if and only if $\Gamma \leq \frac{x_L^* V}{2k}$. No type of player ever has incentive to deviate downwards to a demand $x < x_L^*$ if and only if $\Gamma = \frac{x_L^* V}{2k}$. This ensures that:

$$s^*(x) = \exp\left(\frac{-(x - x_L^*)V}{2k}\right)$$

The complainant's expected utility from the bargaining and litigation subgame is:

$$\begin{aligned} B_C(\pi, n) &= s^*(x^*(\pi, n))[x^*(\pi, n)V - n] + [1 - s^*(x^*(\pi, n))]T_C(\pi, n) \\ &= p(\pi, n)V + n[1 - 2p(\pi, n)] - k + 2k\exp\left(\frac{-[x^*(\pi, n) - x_L^*]V}{2k}\right) \end{aligned}$$

Note that:

$$
\begin{aligned}
\frac{\partial B_C(\pi,n)}{\partial n} &= \frac{\partial p(\pi,n)}{\partial n}(V-2n)+1-2p(\pi,n) \\
&\quad -\exp\left(\frac{-[x^*(\pi,n)-x_L^*]V}{2k}\right)\left[(V-2n)\frac{\partial p(\pi,n)}{\partial n}+2[1-p(\pi,n)]\right] \\
&= \left[1-\exp\left(\frac{-[x^*(\pi,n)-x_L^*]V}{2k}\right)\right]\left[(V-2n)\frac{\partial p(\pi,n)}{\partial n}+1-2p(\pi,n)\right] \\
&\quad -\exp\left(\frac{-[x^*(\pi,n)-x_L^*]V}{2k}\right) \\
&\Rightarrow \lim_{V\to\infty}\frac{\partial B_C(\pi,n)}{\partial n}>0
\end{aligned}
$$

So for large V, the complainant will always want to promote audiences.

Comparative Statics

Analogue of Proposition 1:

$$
\begin{aligned}
\frac{\partial x^*(\pi,n)}{\partial \pi} &= \frac{\partial p(\pi,n)}{\partial \pi}\left[1-\frac{2n}{V}\right]>0 \\
\frac{\partial s^*(x)}{\partial x} &= -\exp\left(\frac{-(x-x_L^*)V}{2k}\right)\left(\frac{V}{2k}\right)<0
\end{aligned}
$$

Analogue of Proposition 2:

$$
\frac{\partial x^*(\pi,n)}{\partial n} = \frac{\partial p(\pi,n)}{\partial n}\left[1-\frac{2n}{V}\right]+\frac{2[1-p(\pi,n)]}{V}>0
$$

Analogue of Proposition 5:

$$
\frac{\partial s^*(x^*(\pi,n))}{\partial n} = -\exp\left(\frac{-[x^*(\pi,n)-x_L^*]V}{2k}\right)\left(\frac{V}{2k}\right)\frac{\partial x^*(\pi,n)}{\partial n}<0
$$

Biased Judicial Rulings with Strategic Partisanship

Model Structure

Suppose that third parties join the disputant with the stronger case.

Let n denote total number of third parties. Let ρ denote the number of third parties who support the complainant. Let σ denote the number of third parties who support the defendant.

Suppose $p(\pi,\rho,\sigma)$ is the probability that the complainant wins the ruling, where $p(\pi,\rho,\sigma)$ is increasing in ρ and decreasing in σ.

General payoffs are:

	Settlement	C wins	D wins
Complainant	$xV - \rho$	$V - n - k$	$n - k$
Defendant	$(1-x)\,V + \rho$	$n - k$	$V - n - k$

Equilibrium Behavior

Case 1: Suppose $\pi > \frac{1}{2}$ (strong complainant)

Then all third parties will join the complainant, so $\rho = n$ and $\sigma = 0$. Payoffs are:

	Settlement	C wins	D wins
Complainant	$xV - n$	$V - n - k$	$n - k$
Defendant	$(1-x)\,V + n$	$n - k$	$V - n - k$

Conditional on x, the defendant will play a mixed strategy $s^*(x)$ if and only if:

$$(1-x)\,V + n = (1-p)\,V - n\,(1-2p) - k \quad \Leftrightarrow \quad x^*(\pi, n) = p + \frac{2n\,(1-p) + k}{V}$$

The complainant's expected utility from x is:

$$\begin{aligned}
EU_C\,(x|\pi, n) &= s(x)\,(xV - n) + [1 - s(x)]\,[pV + n\,(1-2p) - k] \\
\frac{\partial EU_C}{\partial x} &= s(x)V + s'(x)(xV - n) - s'(x)\,[pV + n\,(1-2p) - k] = 0 \\
&\Leftrightarrow \quad x = p + \frac{2n\,(1-p) - k}{V} - \frac{s(x)}{s'(x)}
\end{aligned}$$

So:

$$\begin{aligned}
& p + \frac{2n\,(1-p) + k}{V} = p + \frac{2n\,(1-p) - k}{V} - \frac{s(x)}{s'(x)} \\
\Leftrightarrow \quad & s'(x)\left(\frac{2k}{V}\right) = -s(x) \\
\Rightarrow \quad & s^*(x) = \exp\left(-\frac{xV}{2k} + \Gamma\right)
\end{aligned}$$

Define $x_L^* \equiv \min\{x^*(\pi, n)\}$. The settlement probability is always an interior value if and only if $\Gamma \leq \frac{x_L^* V}{2k}$. No type of player ever has incentive to deviate downwards to a demand $x < x_L^*$ if and only if $\Gamma = \frac{x_L^* V}{2k}$. This ensures that:

$$s^*(x) = \exp\left(\frac{-(x - x_L^*)\,V}{2k}\right)$$

The complainant's expected utility from the bargining-litigation subgame is:

$$\begin{aligned}
B_C\,(\pi, n) &= s^*\,(x^*\,(\pi, n))\,[x^*\,(\pi, n)\,V - n] + [1 - s^*\,(x^*\,(\pi, n))]\,[pV + n\,(1-2p) - k] \\
&= pV + n\,(1-2p) - k + 2k\,\exp\left(-\frac{[x^*\,(\pi, n) - x_L^*]\,V}{2k}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial B_C(\pi,n)}{\partial n} = \frac{\partial B_C(\pi,n)}{\partial \rho} &= \frac{\partial p}{\partial \rho}(V-2n) + 1 - 2p \\
&\quad - \exp\left(-\frac{[x^*(\pi,n) - x_L^*]V}{2k}\right)\left[\frac{\partial p}{\partial \rho}(V-2n) + 2(1-p)\right] \\
&= \left[\frac{\partial p}{\partial \rho}(V-2n) + 1 - 2p\right]\left[1 - \exp\left(-\frac{[x^*(\pi,n) - x_L^*]V}{2k}\right)\right] \\
&\quad - \exp\left(-\frac{[x^*(\pi,n) - x_L^*]V}{2k}\right) > 0 \\
&\Leftrightarrow \frac{\partial p}{\partial \rho}(V-2n) + 1 - 2p(\pi,n) - \frac{\exp\left(-\frac{[x^*(\pi,n)-x_L^*]V}{2k}\right)}{1 - \exp\left(-\frac{[x^*(\pi,n)-x_L^*]V}{2k}\right)} > 0
\end{aligned}$$

So for large V, strong cases will want to promote because $\frac{\partial p}{\partial \rho} > 0$.

Case 2: Suppose $\pi < \frac{1}{2}$ (weak complainant)

Then all third parties will join the defendant, so $\rho = 0$ and $\sigma = n$. Payoffs are:

	Settlement	C wins	D wins
Complainant	xV	$V - n - k$	$n - k$
Defendent	$(1-x)V$	$n - k$	$V - n - k$

Conditional on x, the defendant will play a mixed strategy $s^*(x)$ if and only if:

$$(1-x)V = (1-p)V - n(1-2p) - k \quad \Leftrightarrow \quad x^*(\pi,n) = p + \frac{n(1-2p)+k}{V}$$

The complainant's expected utility from x is:

$$\begin{aligned}
EU_C(x|\pi,n) &= s(x)(xV) + [1 - s(x)][pV + n(1-2p) - k] \\
\frac{\partial EU_C}{\partial x} &= s(x)V + s'(x)(xV) - s'(x)[pV + n(1-2p) - k] = 0 \\
&\Leftrightarrow x = p + \frac{n(1-2p)-k}{V} - \frac{s(x)}{s'(x)}
\end{aligned}$$

So:

$$\begin{aligned}
& p + \frac{n(1-2p)+k}{V} = p + \frac{n(1-2p)-k}{V} - \frac{s(x)}{s'(x)} \\
\Leftrightarrow\ & s'(x)\left(\frac{2k}{V}\right) = -s(x) \\
\Rightarrow\ & s^*(x) = \exp\left(-\frac{xV}{2k} + \Gamma\right)
\end{aligned}$$

Define $x_L^* \equiv \min\{x^*(\pi,n)\}$. The settlement probability is always an interior value if and only if $\Gamma \leq \frac{x_L^* V}{2k}$. No type of player ever has incentive to deviate downwards to a demand $x < x_L^*$ if and only if $\Gamma = \frac{x_L^* V}{2k}$. This ensures that:

$$s^*(x) = \exp\left(\frac{-(x - x_L^*)V}{2k}\right)$$

The complainant's expected utility from the bargining-litigation subgame is:

$$\begin{aligned} B_C(\pi, n) &= s^*(x^*(\pi, n))[x^*(\pi, n)V] + [1 - s^*(x^*(\pi, n))][pV + n(1-2p) - k] \\ &= pV + n(1-2p) - k + 2k \exp\left(-\frac{[x^*(\pi, n) - x_L^*]V}{2k}\right) \end{aligned}$$

$$\begin{aligned} \frac{\partial B_C(\pi, n)}{\partial n} = \frac{\partial B_C(\pi, n)}{\partial \sigma} &= \frac{\partial p}{\partial \sigma}(V - 2n) + 1 - 2p \\ &\quad - \exp\left(-\frac{[x^*(\pi, n) - x_L^*]V}{2k}\right)\left[\frac{\partial p}{\partial \sigma}(V - 2n) + 1 - 2p\right] \\ &= \left[\frac{\partial p}{\partial \sigma}(V - 2n) + 1 - 2p\right]\left[1 - \exp\left(-\frac{[x^*(\pi, n) - x_L^*]V}{2k}\right)\right] < 0 \\ &\Leftrightarrow \frac{\partial p}{\partial \sigma}(V - 2n) + 1 - 2p < 0 \end{aligned}$$

This holds for large V because $\frac{\partial p}{\partial \sigma} < 0$. So weak cases will want to prevent.

Late-Joiners

Model Structure

Continue to assume the same basic model structure and payoffs from the main paper. However, assume that if the defendant rejects the settlement demand, then additional third parties can join the case prior to the trial actually occurring. We assume that Nature chooses the number of these "late-joiners", $\epsilon \sim [0, \epsilon_H]$, according to density function g. We assume that the value of this random variable is not a function of earlier filing decisions. Denote the expected number of late-joiners by: $\bar{\epsilon} \equiv \int_0^{\epsilon_H} \epsilon g(\epsilon) d\epsilon$. Conditional on a realized number of late-joiners, ϵ, payoffs are:

	Early settlement	C wins panel ruling	D wins panel ruling
Complainant	$xV - n$	$V - (n + \epsilon) - k$	$(n + \epsilon) - k$
Defendant	$(1 - x)V + n$	$n + \epsilon - k$	$V - (n + \epsilon) - k$

Prior to the realization of ϵ, the complainant's expected utility from trial is:

$$\begin{aligned} T_C(\pi, n) &= \int_0^{\epsilon_H} [\pi V + (1 - 2\pi)(n + \epsilon) - k]\, g(\epsilon) d\epsilon \\ &= \pi V + (1 - 2\pi)(n + \bar{\epsilon}) - k \end{aligned}$$

and the defendant's expected utility from trial is:

$$\begin{aligned} T_D(\pi,n) &= \int_0^{\epsilon_H} \left[(1-\pi)V-(1-2\pi)(n+\epsilon)-k\right]g(\epsilon)d\epsilon \\ &= (1-\pi)V-(1-2\pi)(n+\bar{\epsilon})-k \end{aligned}$$

Conditional on x, the defendant will play a mixed strategy $s^*(x)$ if and only if:

$$(1-x)V+n=(1-\pi)V-(1-2\pi)(n+\bar{\epsilon})-k \Leftrightarrow x^*(\pi,n)=\pi+\frac{n+(1-2\pi)(n+\bar{\epsilon})+k}{V} \tag{5}$$

This is an interior value for large V. The complainant's expected utility from x is:

$$\begin{aligned} EU_C(x|\pi,n) &= s(x)(xV-n)+[1-s(x)]\,T_C(\pi,n) \\ \Rightarrow \frac{\partial EU_C(x|\pi,n)}{\partial x} &= s(x)V+s'(x)(xV-n)-s'(x)T_C(\pi,n)=0 \\ &\Leftrightarrow \quad x=\frac{T_C(\pi,n)+n}{V}-\frac{s(x)}{s'(x)} \end{aligned} \tag{6}$$

Since both equations must hold simultaneously in equilibrium:

$$\begin{aligned} \pi+\frac{n+(1-2\pi)(n+\bar{\epsilon})+k}{V} &= \frac{T_C(\pi,n)+n}{V}-\frac{s(x)}{s'(x)} \\ \Leftrightarrow \quad \left(\frac{2k}{V}\right)s'(x) &= -s(x) \\ \Rightarrow \quad s^*(x) &= \exp\left(\frac{-xV}{2k}+\Gamma\right) \end{aligned}$$

Define $x_L^* \equiv \min\{x^*(\pi,n)\}$. The settlement probability is always an interior value if and only if $\Gamma \leq \frac{x_L^* V}{2k}$. No type of player ever has incentive to deviate downwards to a demand $x < x_L^*$ if and only if $\Gamma = \frac{x_L^* V}{2k}$. This ensures that:

$$s^*(x)=\exp\left(\frac{-(x-x_L^*)V}{2k}\right)$$

The complainant's expected utility from the bargaining and litigation subgame is:

$$\begin{aligned} B_C(\pi,n) &= s^*(x^*(\pi,n))\,[x^*(\pi,n)V-n]+[1-s^*(x^*(\pi,n))]\,T_C(\pi,n) \\ &= \pi V+(1-2\pi)(n+\bar{\epsilon})-k+2k\,\exp\left(\frac{-[x^*(\pi,n)-x_L^*]V}{2k}\right) \end{aligned}$$

Note that:

$$\begin{aligned} \frac{\partial B_C(\pi,n)}{\partial n} &= 1-2\pi-2(1-\pi)\,\exp\left(\frac{-[x^*(\pi,n)-x_L^*]V}{2k}\right) \\ &\Rightarrow \lim_{V\to\infty}\frac{\partial B_C(\pi,n)}{\partial n}=1-2\pi>0 \Leftrightarrow \pi<\frac{1}{2} \end{aligned}$$

Recall that $\pi_L < \frac{1}{2} < \pi_H$. So for large V, π_L promotes and π_H prevents. Define:

$$\begin{aligned}
\Delta(\pi) &\equiv EU_C\left(\text{promote}|\pi\right) - EU_C\left(\text{prevent}|\pi\right) \\
&= \int_{n_H}^{n_H+\eta} B_C\left(\pi, n\right) f(n|\text{promote}) dn - \int_{n_L}^{n_L+\eta} B_C\left(\pi, n\right) f(n|\text{prevent}) dn \\
&= \frac{1}{\eta}\left[\int_{n_L+\eta}^{n_H+\eta} B_C\left(\pi, n\right) dn - \int_{n_L}^{n_H} B_C\left(\pi, n\right) dn\right]
\end{aligned}$$

By above, $\Delta\left(\pi_L\right) > 0$ and $\Delta\left(\pi_H\right) < 0$ for large V. By the implicit function theorem, there exists $\widetilde{\pi} \in \left(\pi_L, \pi_H\right)$ such that $\Delta\left(\widetilde{\pi}\right) = 0$. To have an equilibrium in which all $\pi < \widetilde{\pi}$ promote and all $\widetilde{\pi} \leq \pi$ prevent, we must show that $\widetilde{\pi}$ is unique:

$$\begin{aligned}
\frac{\partial\Delta(\pi)}{\partial\pi} &= \frac{1}{\eta}\left[\int_{n_L+\eta}^{n_H+\eta} \frac{\partial B_C\left(\pi, n\right)}{\partial\pi} dn - \int_{n_L}^{n_H} \frac{\partial B_C\left(\pi, n\right)}{\partial\pi} dn\right] \\
&= \frac{1}{\eta}\left\{\int_{n_L+\eta}^{n_H+\eta} \left[V - 2\left(n + \overline{\epsilon}\right)\right]\left[1 - \exp\left(\frac{-\left[x^*\left(\pi, n\right) - x_L^*\right] V}{2k}\right)\right] dn\right. \\
&\quad \left. - \int_{n_L}^{n_H} \left[V - 2\left(n + \overline{\epsilon}\right)\right]\left[1 - \exp\left(\frac{-\left[x^*\left(\pi, n\right) - x_L^*\right] V}{2k}\right)\right] dn\right\} \\
&\Rightarrow \lim_{V\to\infty} \frac{\partial\Delta(\pi)}{\partial\pi} < 0
\end{aligned}$$

So for large V, $\widetilde{\pi}$ is unique.

Comparative Statics

Analogue of Proposition 1:

$$\begin{aligned}
\frac{\partial x^*(\pi, n)}{\partial\pi} &= 1 - \frac{2\left(n + \overline{\epsilon}\right)}{V} > 0 \\
\frac{\partial s^*(x)}{\partial x} &= -\exp\left(\frac{-\left(x - x_L^*\right) V}{2k}\right)\left(\frac{V}{2k}\right) < 0
\end{aligned}$$

Analogue of Proposition 2:

$$\frac{\partial x^*(\pi, n)}{\partial n} = \frac{2\left(1 - \pi\right)}{V} > 0$$

Analogues of Propositions 3 and 4 follow from the equilibrium behavior described above.

Analogue of Proposition 5:

$$\frac{\partial s^*\left(x^*(\pi, n)\right)}{\partial n} = -\exp\left(\frac{-\left[x^*\left(\pi, n\right) - x_L^*\right] V}{2k}\right)\left(\frac{V}{2k}\right)\frac{\partial x^*(\pi, n)}{\partial n} < 0$$

Analogue of Proposition 6: Recall that π is distributed according to density f on $[\pi_L, \pi_H]$. Conditional on n:

$$\begin{aligned} s^*\left(x^*\left(\text{promote}|n\right)\right) &= \int_{\pi_L}^{\widetilde{\pi}} s^*\left(x^*\left(\pi, n\right)\right) f(\pi|\text{promote}) d\pi \\ s^*\left(x^*\left(\text{prevent}|n\right)\right) &= \int_{\widetilde{\pi}}^{\pi_H} s^*\left(x^*\left(\pi, n\right)\right) f(\pi|\text{prevent}) d\pi \end{aligned}$$

The equilibrium probability of settlement is decreasing in π:

$$\begin{aligned} \frac{\partial s^*\left(x^*\left(\pi, n\right)\right)}{\partial \pi} &= -\exp\left(\frac{-\left[x^*\left(\pi, n\right) - x_L^*\right] V}{2k}\right)\left(\frac{V}{2k}\right)\frac{\partial x^*(\pi, n)}{\partial \pi} < 0 \\ &\Rightarrow s^*\left(x^*\left(\text{promote}|n\right)\right) > s^*\left(x^*\left(\text{prevent}|n\right)\right) \end{aligned}$$

Number of Third Parties as a Discrete Random Variable

Model Structure

Continue to assume the same basic model structure and payoffs from the main paper. However, assume the following distributions on the number of third parties:

$$\Pr\left(n = \hat{n}\right) = \frac{1}{\eta + 1} \quad \text{for} \quad \begin{cases} \widehat{n} \in \{n_L, n_L + 1, ..., n_L + \eta\} & \text{if the complainant prevents} \\ \widehat{n} \in \{n_H, n_H + 1, ..., n_H + \eta\} & \text{if the complainant promotes} \end{cases}$$

where $n_H < n_L + \eta$.

Equilibrium Behavior

Conditional on x, the defendant will play a mixed strategy if and only if:

$$(1 - x) V + n = (1 - \pi) V - n (1 - 2\pi) - k \quad \Leftrightarrow \quad x^*\left(\pi, n\right) = \pi + \frac{2n\left(1 - \pi\right) + k}{V} \tag{7}$$

This is always an interior solution for large V. Let $T_C(\pi, n)$ denote the complainant's expected utility from litigation. Then the complainant's expected utility from an offer x is:

$$\begin{aligned} EU_C(x|\pi, n) &= s(x)(xV - n) + \left[1 - s(x)\right] T_C(\pi, n) \\ \Rightarrow \frac{\partial EU_C(x|\pi, n)}{\partial x} &= s(x)V + s'(x)(xV - n) - s'(x)T_C(\pi, n) = 0 \\ &\Leftrightarrow x = \frac{T_C(\pi, n) + n}{V} - \frac{s(x)}{s'(x)} \end{aligned} \tag{8}$$

Both equations must hold simultaneously in equilibrium, so:

$$\begin{aligned}\pi+\frac{2n(1-\pi)+k}{V} &= \frac{T_C(\pi,n)+n}{V}-\frac{s(x)}{s'(x)}\\ \Leftrightarrow \left(\frac{2k}{V}\right)s'(x) &= -s(x)\\ \Rightarrow \quad s^*(x) &= \exp\left(\frac{-xV}{2k}+\Gamma\right)\end{aligned}$$

This is always an interior value if and only if $\Gamma \leq \frac{x_L^* V}{2k}$. So we can have a fully separating equilibrium in which:

$$s^*(x)=\begin{cases}1 & \text{if } x<x_L^*\\ \exp\left(\frac{-(x-x_L^*)V}{2k}\right) & \text{if } x\in[x_L^*,x_H^*]\\ 0 & \text{if } x>x_H^*\end{cases}$$

The complainant's expected utility from the bargaining and litigation subgame is:

$$\begin{aligned}B_C(\pi,n) &= s^*(x^*(\pi,n))\,[x^*(\pi,n)V-n]+[1-s^*(x^*(\pi,n))]\,T_C(\pi,n)\\ &= \pi V+n(1-2\pi)-k+2k\,s^*(x^*(\pi,n))\end{aligned}$$

Suppose that $n'<n''$. Define:

$$\begin{aligned}\rho &\equiv B_C(\pi,n'')-B_C(\pi,n')\\ &= (n''-n')(1-2\pi)+2k\,[s^*(x^*(\pi,n''))-s^*(x^*(\pi,n'))]\\ &\Rightarrow \lim_{V\to\infty}\rho=(n''-n')(1-2\pi)\geq 0 \;\Leftrightarrow\; \pi\leq\frac{1}{2}\end{aligned}$$

Recal that $\pi_L<\frac{1}{2}<\pi_H$. So for large V, type π_L wants to promote and type π_H wants to prevent. Define:

$$\begin{aligned}\Delta(\pi) &\equiv EU_C(promote|\pi)-EU_C(prevent|\pi)\\ &= \sum_{n=n_H}^{n_H+\eta}B_C(\pi,n)\Pr(n|promote)-\sum_{n=n_L}^{n_L+\eta}B_C(\pi,n)\Pr(n|prevent)\\ &= \frac{1}{\eta+1}\left[\sum_{n=n_L+\eta}^{n_H+\eta}B_C(\pi,n)-\sum_{n=n_L}^{n_H}B_C(\pi,n)\right]\end{aligned}$$

By above, $\Delta(\pi_L)>0$ and $\Delta(\pi_H)<0$ for large V. By the intermediate value theorem, there exists a type $\widetilde{\pi}\in(\pi_L,\pi_H)$ such that $\Delta(\widetilde{\pi})=0$. To have an equilibrium in which all types $\pi<\widetilde{\pi}$ prevent and all types $\widetilde{\pi}<\pi$ promote, we must show that $\widetilde{\pi}$ is unique:

$$\begin{aligned}\frac{\partial\Delta(\pi)}{\partial\pi} &= \frac{1}{\eta+1}\left[\sum_{n=n_L+\eta}^{n_H+\eta}\frac{\partial B_C(\pi,n)}{\partial\pi}-\sum_{n=n_L}^{n_H}\frac{\partial B_C(\pi,n)}{\partial\pi}\right]\\ &= \frac{1}{\eta+1}\left\{\sum_{n=n_L+\eta}^{n_H+\eta}(V-2n)\left[1-\exp\left(-\frac{[x^*(\pi,n)-x_L^*]V}{2k}\right)\right]\right.\\ &\qquad\left.-\sum_{n=n_L}^{n_H}(V-2n)\left[1-\exp\left(-\frac{[x^*(\pi,n)-x_L^*]V}{2k}\right)\right]\right\}\end{aligned}$$

So $\frac{\partial \Delta(\pi)}{\partial \pi} < 0$ for large V, which means that $\widetilde{\pi}$ is unique.

Comparative Statics

Propositions 1, 3, 4, and 6 follow directly.

For Proposition 2, suppose that $n' < n''$. Then:

$$x^*(\pi, n'') - x^*(\pi, n') = \frac{2(n'' - n')(1-\pi)}{V} > 0$$

For Proposition 5, suppose that $n' < n''$. Then:

$$\begin{aligned} & s^*(x^*(\pi, n'')) < s^*(x^*(\pi, n')) \\ \Leftrightarrow\ & \exp\left(-\frac{[x^*(\pi, n'') - x_L^*]V}{2k}\right) < \exp\left(-\frac{[x^*(\pi, n') - x_L^*]V}{2k}\right) \\ \Leftrightarrow\ & \frac{[x^*(\pi, n') - x_L^*]V}{2k} < \frac{[x^*(\pi, n'') - x_L^*]V}{2k} \\ \Leftrightarrow\ & x^*(\pi, n') < x^*(\pi, n'') \end{aligned}$$

This holds by Proposition 2.