1 Alternative Punishment Mechanisms

The one-period utility functions of the home and foreign government—$W$ and $W^*$, respectively—are as follows:

\[
W(t, \tau, a) = a u(t) - t - u(\tau) \\
W^*(t, \tau, \alpha) = \alpha u(\tau) - \tau - u(t)
\]

Losses are:

\[
L(\tau) = W(t, t_B, a) - W(t, \tau, a) = u(\tau) - u(t_B) \\
L^*(t) = W^*(t_B, \tau, \alpha) - W^*(t, \tau, \alpha) = u(t) - u(t_B)
\]

Let $\chi_P$ denote the continuation payoff from the punishment that occurs if at least one player defects. Assume that $\chi_P$ is not a function of the specific value of the defection tariff.

Let $\chi_C$ denote the continuation payoff if the treaty remains in effect (neither player defects).

Recall that $a, \alpha \sim_{iid} U[1, A]$ for large $A$, $u' > 0$, and $u'' < 0$.

1.1 Optimal Tariffs

The home country’s expected utility from violating the binding and not paying the fine (defection) is:

\[
EU(D|t, a) = a u(t) - t - \int_1^A u(\tau(\alpha)) dH(\alpha) + \int_{\alpha_2}^{\alpha_D} \sigma L(\tau(\alpha)) dH(\alpha) + \delta \chi_P
\]

So the optimal defection tariff solves:

\[
\frac{\partial EU(D|t, a)}{\partial t} = a u'(t) - 1 = 0 \\
\iff u'(t) = \frac{1}{a} \iff t_D(a) = u'^{-1}\left(\frac{1}{a}\right)
\]

This violates the binding iff:

\[
t_D(a) = u'^{-1}\left(\frac{1}{a}\right) > t_B \iff \frac{1}{a} < u'(t_B) \iff a > \frac{1}{u'(t_B)} = a_B
\]
The home country’s expected utility from violating the binding and paying the fine (settlement) is:

\[
EU(S|t, a) = au(t) - t - \sigma L^*(t) - \int_1^A u(\tau(\alpha))dH(\alpha) + \int_{\alpha_S}^{\alpha_D} \sigma L(\tau(\alpha))dH(\alpha) + H(\alpha_D)\delta\beta\chi_C + [1 - H(\alpha_D)]\delta\chi_P
\]

So the optimal settlement tariff solves:

\[
\frac{\partial EU(S|t, a)}{\partial t} = au'(t) - 1 - \sigma u'(t) = 0
\]

\[\iff u'(t) = \frac{1}{a - \sigma} \iff t_S(a) = u^{-1}\left(\frac{1}{a - \sigma}\right)\]

This violates the binding iff:

\[t_S(a) = u^{-1}\left(\frac{1}{a - \sigma}\right) > t_B \iff \frac{1}{a - \sigma} < u'(t_B)\]

\[\iff a > \frac{1}{u'(t_B)} + \sigma \equiv a_S\]

Note that: \(t_S(a) < t_D(a)\) for all \(a\).

The optimal cooperative tariff is:

\[t_B(a) = \begin{cases} t_D(a) & \text{if } a < a_B \\ t_B & \text{if } a_B \leq a \end{cases}\]

### 1.2 Equilibrium Regions

The home country’s expected utility from actions \(C, S\), and \(D\) given optimal tariff levels are:

\[
EU(C|t_B(a), a) = au(t_B(a)) - t_B(a) - \int_1^A u(\tau(\alpha))dH(\alpha) + \int_{\alpha_S}^{\alpha_D} \sigma L(\tau(\alpha))dH(\alpha) + H(\alpha_D)\delta\beta\chi_C + [1 - H(\alpha_D)]\delta\chi_P
\]

\[
EU(S|t_S(a), a) = au(t_S(a)) - t_S(a) - \sigma L^*(t_S(a)) - \int_1^A u(\tau(\alpha))dH(\alpha) + \int_{\alpha_S}^{\alpha_D} \sigma L(\tau(\alpha))dH(\alpha) + H(\alpha_D)\delta\beta\chi_C + [1 - H(\alpha_D)]\delta\chi_P
\]

\[
EU(D|t_D(a), a) = au(t_D(a)) - t_D(a) - \int_1^A u(\tau(\alpha))dH(\alpha) + \int_{\alpha_S}^{\alpha_D} \sigma L(\tau(\alpha))dH(\alpha) + \delta\chi_P
\]

To compare utility from actions \(C\) and \(S\), define for \(a_S \leq a\):

\[
\hat{\Delta}(a) = EU(C|t_B(a), a) - EU(S|t_S(a), a) = au(t_B) - t_B - au(t_S(a)) + t_S(a) + \sigma L^*(t_S(a))
\]

Note that \(t_S(a) = t_B\), so \(\hat{\Delta}(a_S) = 0\). Also:

\[
\frac{\partial \hat{\Delta}}{\partial a} = u(t_B) - u(t_S(a)) - (a - \sigma)u'(t_S(a))\frac{\partial t_S(a)}{\partial a} + \frac{\partial t_S(a)}{\partial a} = u(t_B) - u(t_S(a)) < 0
\]
So $S$ strictly dominates $C$ for all $a_S < a$.

To compare utility from actions $S$ and $D$, define for $a_S \leq a$:

$$
\bar{\Delta}(a) = EU(S|t_S(a), a) - EU(D|t_D(a), a) = a u(t_S(a)) - t_S(a) - \sigma L^*(t_S(a)) - a u(t_D(a)) + t_D(a) + \delta H(a_D)(\beta \chi C - \chi P)
$$

So:

$$
\frac{\partial \bar{\Delta}}{\partial a} = (a - \sigma) u'(t_S(a)) \frac{\partial t_S(a)}{\partial a} - \frac{\partial t_S(a)}{\partial a} + u(t_S(a)) + \frac{\partial t_D(a)}{\partial a} - a u'(t_D(a)) \frac{\partial t_D(a)}{\partial a} - t_D(a) = u(t_S(a)) - u(t_D(a)) < 0
$$

So $D$ strictly dominates $S$ for sufficiently large values of $a$. By symmetry, indifference point $a_D$ is implicitly defined by:

$$
\lambda = a_D \left[ u(t_S(a_D)) - u(t_D(a_D)) \right] + t_D(a_D) - t_S(a_D) - \sigma L^*(t_S(a_D)) + \delta H(a_D)(\beta \chi C - \chi P) = 0
$$

The equilibrium exists iff: $\bar{\Delta}(a_S) > 0$.

### 1.3 Continuation Values

Let $t_E(a)$ denote equilibrium tariffs when the institution is in place.

The continuation payoff for home from the treaty being in effect is:

$$
\chi_C = \int_0^A \left[ au(t_E(a)) - t_E(a) \right] dH(a) - \sigma \int_{a_S}^{a_D} L^*(t_E(a)) dH(a) - \int_1^A u(t_E(a)) dH(a) + \sigma \int_{a_S}^{a_D} L^*(t_E(a)) dH(a) + \delta H(a_D)^2 \beta \chi C + \delta \left[ 1 - H(a_D)^2 \right] \chi P
$$

where

$$
\Psi = \int_1^A \left[ (a - 1) u(t_E(a)) - t_E(a) \right] dH(a) + \delta \left[ 1 - H(a_D)^2 \right] \chi P
$$

### 1.4 Comparative Statics

**Full Compliance**

Recall that the binding is not violated if $a < a_s = \frac{1}{u'(t_B)} + \sigma$. So the probability that the binding is not violated is $H(a_S)$.

$$
\frac{\partial a_S}{\partial t_B} = \frac{-u''(t_B)}{[u'(t_B)]^2} > 0 \quad \text{and} \quad \frac{\partial a_S}{\partial \sigma} = 1 > 0
$$

**Stability**
The institution is stable if \( a < a_D \). By the implicit function theorem:

\[
\frac{\partial a_D}{\partial t_B} = \frac{\lambda_{t_B}}{\lambda_{a_D}} \quad \text{and} \quad \frac{\partial a_D}{\partial \sigma} = -\frac{\lambda_{t}}{\lambda_{a_D}}
\]

Then for large \( A \):

\[
\begin{align*}
\lambda_{a_D} &= (a_D - \sigma) u'(t_S(a_D)) \left(\frac{\partial t_S(a_D)}{\partial a_D} - \frac{\partial t_S(a_D)}{\partial a_D} - a_D u'(t_D(a_D)) \frac{\partial t_D(a_D)}{\partial a_D} + \frac{\partial t_D(a_D)}{\partial a_D}\right) \\
&+ u(t_S(a_D)) - u(t_D(a_D)) + \delta H(a_D) \frac{\partial (\beta\chi_C - \chi_P)}{\partial a_D} + \delta h(a_D) (\beta\chi_C - \chi_P) \\
&= u(t_S(a_D)) - u(t_D(a_D)) + \delta H(a_D) \frac{\partial (\beta\chi_C - \chi_P)}{\partial a_D} + \delta h(a_D) (\beta\chi_C - \chi_P) < 0
\end{align*}
\]

\[
\begin{align*}
\lambda_{t_B} &= \sigma u'(t_B) + \delta H(a_D) \frac{\partial (\beta\chi_C - \chi_P)}{\partial t_B} > 0 \\
\lambda_{\sigma} &= (a_D - \sigma) u'(t_S(a_D)) \frac{\partial t_S(a_D)}{\partial \sigma} - \frac{\partial t_S(a_D)}{\partial \sigma} - [u(t_S(a_D)) - u(t_B)] + \delta H(a_D) \frac{\partial (\beta\chi_C - \chi_P)}{\partial \sigma} \\
&= - [u(t_S(a_D)) - u(t_B)] + \delta H(a_D) \frac{\partial (\beta\chi_C - \chi_P)}{\partial \sigma} < 0
\end{align*}
\]

So:

\[
\frac{\partial a_D}{\partial t_B} > 0 \quad \text{and} \quad \frac{\partial a_D}{\partial \sigma} < 0
\]

### Depth versus Rigidity

Recall that \( \chi_C \) is the expected utility of a state from being a member of the cooperative regime. In equilibrium, \( \lambda = 0 \). So for any pair \((t_B, \sigma)\):

\[
\chi_C = a_D \left[ u(t_D(a_D)) - u(t_S(a_D)) \right] - t_D(a_D) + t_S(a_D) + \sigma L^* (t_S(a_D)) + \frac{\chi_P}{\beta}
\]

The two first-order conditions on the optimal pair \((t_B, \sigma)\) are:

\[
\begin{align*}
\frac{d\chi_C}{dt_B} &= \frac{\partial \chi_C}{\partial t_B} + \frac{\partial \chi_C}{\partial a_D} \frac{\partial a_D}{\partial t_B} = 0 \\
\frac{d\chi_C}{d\sigma} &= \frac{\partial \chi_C}{\partial \sigma} + \frac{\partial \chi_C}{\partial a_D} \frac{\partial a_D}{\partial \sigma} = 0
\end{align*}
\]

This implies that:

\[
\frac{\partial \chi_C}{\partial t_B} = \frac{\partial \chi_C}{\partial \sigma} \quad \frac{\partial a_D}{\partial t_B} = \frac{\partial a_D}{\partial \sigma}
\]

\[
\left( \frac{\partial \chi_C}{\partial t_B} \right) \left( \frac{\partial a_D}{\partial \sigma} \right) = \frac{\partial a_D}{\partial t_B}
\]

So for any pair \((t_B, \sigma)\) that generates \( \chi_C (t_B, \sigma) = \chi^* \):
\[
\frac{dt_B}{d\sigma} = \frac{d\chi_C}{dt_B} = -\left( \frac{\partial \chi_C}{\partial \sigma} + \frac{\partial \chi_C}{\partial a_D} \frac{\partial a_D}{\partial \sigma} \right)
\]

where:

\[
\frac{\partial \chi_C}{\partial \sigma} = \frac{1}{\delta \beta H(a_D)} \left[ \frac{\partial \chi_C}{\partial \sigma} \left( t_S(a_D) \frac{\partial t_S(a_D)}{\partial \sigma} + \frac{\partial t_S(a_D)}{\partial \sigma} + \sigma u'(t_S(a_D)) \frac{\partial t_S(a_D)}{\partial \sigma} \right) \right]
\]

\[
\frac{\partial \chi_C}{\partial t_B} = \frac{\partial \chi_C}{\partial \sigma} \left[ t_S(a_D) \frac{\partial \chi_C}{\partial \sigma} + \frac{\partial \chi_C}{\partial a_D} \frac{\partial a_D}{\partial \sigma} \right]
\]

So:

\[
\frac{dt_B}{d\sigma} = -\frac{\partial \chi_C}{\partial \sigma} + \frac{\partial \chi_C}{\partial a_D} \frac{\partial a_D}{\partial \sigma} + \sigma u'(t_B) \frac{\partial \chi_C}{\partial t_B}
\]

\[
\frac{dt_B}{d\sigma} = \frac{L^*(t_S(a_D)) + \delta H(a_D) \frac{\partial \chi_C}{\partial t_B}}{\delta u'(t_B) - \delta H(a_D) \frac{\partial \chi_C}{\partial t_B}} > 0 \text{ for small } A
\]

2 Asymmetric Type Distributions

Assume that home country type, \( a \), is distributed according to distribution function \( H(a) \). Denote home continuation payoffs by \( \chi_N \) and \( \chi_C \).

Assume that foreign country type, \( \alpha \), is distributed according to distribution function \( F(\alpha) \). Denote foreign continuation payoffs \( \chi_N^* \) and \( \chi_C^* \).

The one-period utility functions of the home and foreign government—\( W \) and \( W^* \), respectively—are as follows:

\[
W(t, \tau, a) = a u(t) - t - u(\tau)
\]

\[
W^*(t, \tau, \alpha) = \alpha u(\tau) - \tau - u(t)
\]
Losses are:

\[ L(\tau) = W(t, \tau_B, a) - W(t, \tau, a) = u(\tau) - u(\tau_B) \]
\[ L^*(t) = W^*(t_B, \tau, a) - W^*(t, \tau, a) = u(t) - u(t_B) \]

2.1 Optimal Tariffs

Home

The home country’s expected utility from from violating the binding and not paying compensation (defection) is:

\[ U(D|t, a) = a u(t) - t - \int u(\tau(\alpha)) \ dF(\alpha) + \int_{\alpha_D}^{\alpha_S} \sigma L(\tau(\alpha)) \ dF(\alpha) + \delta \chi_N \]

So the optimal defection tariff solves:

\[ \frac{\partial U(D|t, a)}{\partial t} = a u'(t) - 1 = 0 \]
\[ \Leftrightarrow u'(t) = \frac{1}{a} \Leftrightarrow t_D(a) = u'^{-1}\left(\frac{1}{a}\right) \]

This violates the home binding iff:

\[ t_D(a) = u'^{-1}\left(\frac{1}{a}\right) > t_B \Leftrightarrow \frac{1}{a} < u'(t_B) \Leftrightarrow a > \frac{1}{u'(t_B)} \equiv a_B \]

The home country’s expected utility from violating the binding and paying compensation (settlement) is:

\[ U(S|t, a) = a u(t) - t - \sigma L^*(t) - \int u(\tau(\alpha)) \ dF(\alpha) + \int_{\alpha_D}^{\alpha_S} \sigma L(\tau(\alpha)) \ dF(\alpha) + F(\alpha_D) \delta \beta \chi_C + [1 - F(\alpha_D)] \delta \chi_N \]

So the optimal settlement tariff solves:

\[ \frac{\partial U(S|t, a)}{\partial t} = a u'(t) - 1 - \sigma u'(t) = 0 \]
\[ \Leftrightarrow u'(t) = \frac{1}{a - \sigma} \Leftrightarrow t_S(a) = u'^{-1}\left(\frac{1}{a - \sigma}\right) \]

This violates the home binding iff:

\[ t_S(a) = u'^{-1}\left(\frac{1}{a - \sigma}\right) > t_B \Leftrightarrow \frac{1}{a - \sigma} < u'(t_B) \]
\[ \Leftrightarrow a > \frac{1}{u'(t_B)} + \sigma \equiv a_S \]
Note that: $t_S(a) < t_D(a)$ for all $a$. The optimal cooperative tariff is:

$$t_B(a) = \begin{cases} 
   t_D(a) & \text{if } a < a_B \\
   t_B & \text{if } a_B \leq a 
\end{cases}$$

**Foreign**

The foreign country’s expected utility from from violating the binding and not paying compensation (defection) is:

$$U^*(D|\tau, \alpha) = \alpha u(\tau) - \tau - \int u(t(a)) \, dH(a) + \int_{a_S}^{a_D} \sigma L^*(t(a)) \, dH(a) + \delta \chi_N$$

So the optimal defection tariff solves:

$$\frac{\partial U^*(D|\tau, \alpha)}{\partial \tau} = \alpha u'(\tau) - 1 = 0$$

$$\iff u'(\tau) = \frac{1}{\alpha} \iff \tau_D(\alpha) = u'^{-1}\left(\frac{1}{\alpha}\right)$$

This violates the foreign binding iff:

$$\tau_D(\alpha) = u'^{-1}\left(\frac{1}{\alpha}\right) > \tau_B \iff \frac{1}{\alpha} < u'(\tau_B) \iff \alpha > \frac{1}{u'(\tau_B)} \equiv \alpha_B$$

The foreign country’s expected utility from violating the binding and paying compensation (settlement) is:

$$U^*(S|\tau, \alpha) = \alpha u(\tau) - \tau - \sigma L(\tau) - \int u(t(a)) \, dH(a) + \int_{a_S}^{a_D} \sigma L^*(t(a)) \, dH(a) + H(a_D) \delta \beta \chi_C + \left[1 - H(a_D)\right] \delta \chi_N$$

So the optimal settlement tariff solves:

$$\frac{\partial U^*(S|\tau, \alpha)}{\partial \tau} = \alpha u'(\tau) - 1 - \sigma u'(\tau) = 0$$

$$\iff u'(\tau) = \frac{1}{\alpha - \sigma} \iff \tau_S(\alpha) = u'^{-1}\left(\frac{1}{\alpha - \sigma}\right)$$

This violates the foreign binding iff:

$$\tau_S(\alpha) = u'^{-1}\left(\frac{1}{\alpha - \sigma}\right) > \tau_B \iff \frac{1}{\alpha - \sigma} < u'(\tau_B) \iff \alpha > \frac{1}{u'(\tau_B)} + \sigma \equiv \alpha_S$$

Note that: $\tau_S(\alpha) < \tau_D(\alpha)$ for all $\alpha$. The optimal cooperative tariff is:

$$\tau_B(\alpha) = \begin{cases} 
   \tau_D(\alpha) & \text{if } \alpha < \alpha_B \\
   \tau_B & \text{if } \alpha_B \leq \alpha 
\end{cases}$$
2.2 Equilibrium Regions

Home

The home country’s expected utility from actions $C$, $S$, and $D$ given tariff levels above are:

\[
U(C|t_B(a), a) = au(t_B(a)) - t_B(a) - \int u(\tau(\alpha))dF(\alpha) + \int_0^{\alpha_D} \sigma L(\tau(\alpha))dF(\alpha) + F(\alpha_D)\delta\beta\chi + [1 - F(\alpha_D)]\delta\chi_N
\]

\[
U(S|t_S(a), a) = au(t_S(a)) - t_S(a) - \sigma L^*(t_S(a)) - \int u(\tau(\alpha))dF(\alpha) + \int_0^{\alpha_D} \sigma L(\tau(\alpha))dF(\alpha) + F(\alpha_D)\delta\beta\chi + [1 - F(\alpha_D)]\delta\chi_N
\]

\[
U(D|t_D(a), a) = au(t_D(a)) - t_D(a) - \int u(\tau(\alpha))dF(\alpha) + \int_0^{\alpha_D} \sigma L(\tau(\alpha))dF(\alpha) + \delta\chi_N
\]

To compare home utility from actions $C$ and $S$, define for $a_S \leq a$:

\[
\hat{\Delta}(a) = U(C|t_B(a), a) - U(S|t_S(a), a)
= au(t_B) - t_B - au(t_S(a)) + t_S(a) + \sigma L^*(t_S(a))
\]

Note that $t_S(a_S) = t_B$, so $\hat{\Delta}(a_S) = 0$. Also:

\[
\frac{\partial\hat{\Delta}}{\partial a} = u(t_B) - u(t_S(a)) - (a - \sigma)u'(t_S(a)) \frac{\partial t_S(a)}{\partial a} + \frac{\partial t_S(a)}{\partial a}
= u(t_B) - u(t_S(a)) < 0
\]

So $S$ strictly dominates $C$ for all $a_S < a$.

To compare home utility from actions $S$ and $D$, define for $a_S \leq a$:

\[
\tilde{\Delta}(a) = U(S|t_S(a), a) - U(D|t_D(a), a)
= au(t_S(a)) - t_S(a) - \sigma L^*(t_S(a))
- au(t_D(a)) + t_D(a) + \sigma F(\alpha_D)(\beta\chi_C - \chi_N)
\]

So:

\[
\frac{\partial\tilde{\Delta}}{\partial a} = (a - \sigma)u'(t_S(a)) \frac{\partial t_S(a)}{\partial a} - \frac{\partial t_S(a)}{\partial a} + u(t_S(a))
+ \frac{\partial t_D(a)}{\partial a} - au'(t_D(a)) \frac{\partial t_D(a)}{\partial a} - u(t_D(a))
= u(t_S(a)) - u(t_D(a)) < 0
\]

So $D$ strictly dominates $S$ for sufficiently large values of $a$.

Indifference point $a_D$ is implicitly defined by:

\[
\lambda = a_D [u(t_S(a_D)) - u(t_D(a_D))] + t_D(a_D) - t_S(a_D)
- \sigma L^*(t_S(a_D)) + \sigma F(\alpha_D)(\beta\chi_C - \chi_N) = 0
\]
The equilibrium exists iff: \( \Delta (a_S) > 0. \)

**Foreign**

The foreign country’s expected utility from actions \( C, S, \) and \( D \) given tariff levels above are:

\[
U^* (C | \tau_B (\alpha), \alpha) = \alpha u (\tau_B (\alpha)) - \tau_B (\alpha) - \int u(t(a))dH(a) + \int \sigma L^* (t(a))dH(a) \\
+ H (a_D) \delta \beta \chi^*_C + [1 - H (a_D)] \delta \chi^*_N
\]

\[
U^* (S | \tau_S (\alpha), \alpha) = \alpha u (\tau_S (\alpha)) - \tau_S (\alpha) - \sigma L (\tau_S (\alpha)) - \int u(t(a))dH(a) + \int \sigma L^* (t(a))dH(a) \\
+ H (a_D) \delta \beta \chi^*_C + [1 - H (a_D)] \delta \chi^*_N
\]

\[
U^* (D | \tau_D (\alpha), \alpha) = \alpha u (\tau_D (\alpha)) - \tau_D (\alpha) - \int u(t(a))dH(a) + \int \sigma L^* (t(a))dH(a) + \delta \chi^*_N
\]

To compare foreign utility from actions \( C \) and \( S \), define for \( \alpha_S \leq \alpha \):

\[
\hat{\Delta}^*(\alpha) = U^* (C | \tau_B (\alpha), \alpha) - U^* (S | \tau_S (\alpha), \alpha) \\
= \alpha u (\tau_B) - \tau_B - \alpha u (\tau_S (\alpha)) + \tau_S (\alpha) + \sigma L (\tau_S (\alpha))
\]

Note that \( \tau_S (\alpha_S) = \tau_B \), so \( \hat{\Delta}^* (\alpha_S) = 0 \). Also:

\[
\frac{\partial \hat{\Delta}^*}{\partial \alpha} = u (\tau_B) - u (\tau_S (\alpha)) - (\alpha - \sigma) u' (\tau_S (\alpha)) \frac{\partial \tau_S (\alpha)}{\partial \alpha} + \frac{\partial \tau_S (\alpha)}{\partial \alpha} \\
= u (\tau_B) - u (\tau_S (\alpha)) < 0
\]

So \( S \) strictly dominates \( C \) for all \( \alpha_S < \alpha \).

To compare foreign utility from actions \( S \) and \( D \), define for \( \alpha_S \leq \alpha \):

\[
\hat{\Delta}^* (\alpha) = U^* (S | \tau_S (\alpha), \alpha) - U^* (D | \tau_D (\alpha), \alpha) \\
= \alpha u (\tau_S (\alpha)) - \tau_S (\alpha) - \sigma L (\tau_S (\alpha)) \\
- \alpha u (\tau_D (\alpha)) + \tau_D (\alpha) + \delta H (a_D) (\beta \chi^*_C - \chi^*_N)
\]

So:

\[
\frac{\partial \hat{\Delta}^*}{\partial \alpha} = (\alpha - \sigma) u' (\tau_S (\alpha)) \frac{\partial \tau_S (\alpha)}{\partial \alpha} - \frac{\partial \tau_S (\alpha)}{\partial \alpha} + u (\tau_S (\alpha)) \\
+ \frac{\partial \tau_D (\alpha)}{\partial \alpha} - \alpha u' (\tau_D (\alpha)) \frac{\partial \tau_D (\alpha)}{\partial \alpha} - u (\tau_D (\alpha)) \\
= u (\tau_S (\alpha)) - u (\tau_D (\alpha)) < 0
\]

So \( D \) strictly dominates \( S \) for sufficiently large values of \( \alpha \).
Indifference point $\alpha_D$ is implicitly defined by:

$$
\lambda^* = \alpha_D \left[ u(\tau_S(\alpha_D)) - u(\tau_D(\alpha_D)) \right] + \tau_D(\alpha_D) - \tau_S(\alpha_D) \\
- \sigma L(\tau_S(\alpha_D)) + \delta H(a_D) (\beta \chi^* - \chi^*_N) = 0
$$

The equilibrium exists iff $\Delta^*(\alpha_S) > 0$.

### 2.3 Continuation Values

Let $t_E(a)$ and $\tau_E(\alpha)$ denote equilibrium tariffs of the home and foreign country, respectively, when the institution is in place.

If the institution does not exist, then the home and foreign country choose $t_D(a)$ and $\tau_D(\alpha)$ in every time period. This yields anarchy continuation payoffs:

$$
\chi^*_N = \frac{1}{1 - \delta} \left\{ \int [\alpha u(\tau_D(\alpha)) - \tau_D(\alpha)] dH(\alpha) - \int u(\tau_E(\alpha)) dF(\alpha) \right\}
$$

$$
\chi^*_N = \frac{1}{1 - \delta} \left\{ \int [\alpha u(\tau_D(\alpha)) - \tau_D(\alpha)] dF(\alpha) - \int u(t_D(a)) dH(a) \right\}
$$

**Home**

The continuation payoff for home from the treaty being in effect is:

$$
\chi^*_C = \int [a u(t_E(a)) - t_E(a)] dH(a) - \sigma \int \alpha^*_D L^*(t_E(a)) dH(a) - \int u(\tau_E(\alpha)) dF(\alpha)
\\
+ \sigma \int L(\tau_E(\alpha)) dF(\alpha) + \delta H(a_D) F(\alpha_D) \beta \chi^* - \chi^*_N \chi_N
\\
= \frac{1 - \delta \beta H(a_D) F(\alpha_D)}{1 - \delta \beta H(a_D) F(\alpha_D)}
$$

where

$$
\Psi = \int [a u(t_E(a)) - t_E(a)] dH(a) - \sigma \int \alpha^*_D L^*(t_E(a)) dH(a) - \int u(\tau_E(\alpha)) dF(\alpha)
\\
+ \sigma \int L(\tau_E(\alpha)) dF(\alpha) + \delta [1 - H(a_D) F(\alpha_D)] \chi^*_N
$$

**Foreign**

The continuation payoff for foreign from the treaty being in effect is:
\[\chi_C^* = \int [\alpha u(\tau_E(\alpha)) - \tau_E(\alpha)] dF(\alpha) - \sigma \int_{\alpha_S}^{\alpha_D} L(\tau_E(\alpha)) dF(\alpha) - \int u(t_E(\alpha)) dH(\alpha)\]
\[+ \sigma \int_{\alpha_S}^{\alpha_D} L^*(t_E(\alpha)) dH(\alpha) + \delta H(\alpha_D) F(\alpha_D) \beta \chi_C^* + \delta [1 - H(\alpha_D) F(\alpha_D)] \chi_N^* \]
\[= \frac{\Psi^*}{1 - \delta \beta H(\alpha_D) F(\alpha_D)}\]

where \(\Psi^* = \int [\alpha u(\tau_E(\alpha)) - \tau_E(\alpha)] dF(\alpha) - \sigma \int_{\alpha_S}^{\alpha_D} L(\tau_E(\alpha)) dF(\alpha) - \int u(t_E(\alpha)) dH(\alpha)\]
\[+ \sigma \int_{\alpha_S}^{\alpha_D} L^*(t_E(\alpha)) dH(\alpha) + \delta [1 - H(\alpha_D) F(\alpha_D)] \chi_N^* \]

### 2.4 Comparative Statics

**Full Compliance**

Recall that the home country does not violate its binding if \(a \leq a_S = \frac{1}{u'(t_B)} + \sigma\).

\[
\frac{\partial a_S}{\partial t_B} = \frac{-u''(t_B)}{|u'(t_B)|^2} > 0 \quad \text{and} \quad \frac{\partial a_S}{\partial \sigma} = 1 > 0
\]

Recall that the foreign country does not violate its binding if \(\alpha \leq \alpha_S = \frac{1}{u'(\tau_B)} + \sigma\).

\[
\frac{\partial \alpha_S}{\partial \tau_B} = \frac{-u''(\tau_B)}{|u'(\tau_B)|^2} > 0 \quad \text{and} \quad \frac{\partial \alpha_S}{\partial \sigma} = 1 > 0
\]

**Stability**

The cutpoints \((a_D, \alpha_D)\) are implicitly defined by the system of equations:

\[
\lambda(a_D, \alpha_D) = 0
\]
\[
\lambda^*(a_D, \alpha_D) = 0
\]

By Cramer’s Rule:

\[
\frac{\partial a_D}{\partial t_B} = \begin{vmatrix} -\lambda_B & \lambda_{a_D} \\ -\lambda^*_B & \lambda^*_{a_D} \end{vmatrix} \quad \text{and} \quad \frac{\partial a_D}{\partial \sigma} = \begin{vmatrix} -\lambda_{a_D} & \lambda_{a_D} \\ -\lambda^*_{a_D} & \lambda^*_{a_D} \end{vmatrix}
\]
\[
\frac{\partial \alpha_D}{\partial \tau_B} = \begin{vmatrix}
-\lambda_{\tau_B}^* & \lambda_{\alpha_D}^* \\
-\lambda_{\tau_B} & \lambda_{\alpha_D}
\end{vmatrix}
\text{ and } \frac{\partial \alpha_D}{\partial \sigma} = \begin{vmatrix}
-\lambda_{\sigma}^* & \lambda_{\alpha_D}^* \\
-\lambda_{\sigma} & \lambda_{\alpha_D}
\end{vmatrix}
\]

where:

\[
\lambda_{\alpha_D} = u(t_S(a_D)) - u(t_D(a_D)) + \delta F(\alpha_D) \beta \frac{\partial \chi_C}{\partial \alpha_D},
\lambda_{\alpha_D}^* = \delta f(\alpha_D)(\beta \chi_C - \chi_N),
\lambda_{\tau_B} = \sigma u'(t_B) + \delta F(\alpha_D) \beta \frac{\partial \chi_C}{\partial \tau_B},
\lambda_{\tau_B}^* = \sigma u'(t_B) + \delta H(a_D) \beta \frac{\partial \chi_C^*}{\partial \tau_B},
\lambda_{\sigma} = -L^*(t_S(a_D)) + \delta F(\alpha_D) \beta \frac{\partial \chi_C^*}{\partial \sigma},
\lambda_{\sigma}^* = \delta H(a_D) \beta \frac{\partial \chi_C^*}{\partial \sigma}
\]

**Comparative statics on \(\alpha_D\)**

As \(h(a_D), H(a_D), f(\alpha_D), F(\alpha_D)\) grow small:

\[
\begin{vmatrix}
\lambda_{\alpha_D} & \lambda_{\alpha_D}^* \\
\lambda_{\alpha_D} & \lambda_{\alpha_D}^*
\end{vmatrix} = \lambda_{\alpha_D} \lambda_{\alpha_D}^* - \lambda_{\alpha_D} \lambda_{\alpha_D}^*
\]

\[
= \left[ u(t_S(a_D)) - u(t_D(a_D)) + \delta F(\alpha_D) \beta \frac{\partial \chi_C}{\partial \alpha_D} \right] \times \left[ u(t_S(a_D)) - u(t_D(a_D)) + \delta H(a_D) \beta \frac{\partial \chi_C^*}{\partial \alpha_D} \right]
- \left[ \delta F(\alpha_D) \beta \left( \frac{\partial \chi_C}{\partial \alpha_D} \right) + \delta f(\alpha_D)(\beta \chi_C - \chi_N) \right] \times \left[ \delta H(a_D) \beta \left( \frac{\partial \chi_C^*}{\partial \alpha_D} \right) + \delta H(a_D) (\beta \chi_C^* - \chi_N^*) \right]
\rightarrow [u(t_S(a_D)) - u(t_D(a_D))] \times [u(t_S(a_D)) - u(t_D(a_D))] > 0
\]
\[
\begin{vmatrix}
\lambda_B & \lambda_D \\
\lambda_D^* & \lambda_D \\
\end{vmatrix} = 
\begin{vmatrix}
-\lambda_B & \lambda_D \\
-\lambda_B^* & \lambda_D^* \\
\end{vmatrix}
= -\lambda_B \lambda_D^* + \lambda_D \lambda_B^*
\]

\[
\begin{aligned}
\partial_D \frac{\partial}{\partial t} &= 
\begin{vmatrix}
-\lambda_B & \lambda_D \\
-\lambda_B^* & \lambda_D^* \\
\end{vmatrix}
\rightarrow 
-\sigma u'(t_B) [u(\tau_S(\alpha_D)) - u(\tau_D(\alpha_D))] \\
&\quad \frac{[u(t_S(\alpha_D)) - u(t_D(\alpha_D))] \times [u(\tau_S(\alpha_D)) - u(\tau_D(\alpha_D))]}\
&\quad > 0
\end{aligned}
\]

\[
\begin{aligned}
\partial_D \frac{\partial}{\partial \sigma} &= 
\begin{vmatrix}
-\lambda_D & \lambda_D \\
-\lambda_D^* & \lambda_D^* \\
\end{vmatrix}
\rightarrow 
L^* [t_S(\alpha_D)] [u(\tau_S(\alpha_D)) - u(\tau_D(\alpha_D))] \\
&\quad \frac{[u(t_S(\alpha_D)) - u(t_D(\alpha_D))] \times [u(\tau_S(\alpha_D)) - u(\tau_D(\alpha_D))]}\
&\quad < 0
\end{aligned}
\]

**Comparative Statics on** \(\alpha_D\)

As \(h(\alpha_D), H(\alpha_D), f(\alpha_D), F(\alpha_D)\) grow small:

\[
\begin{vmatrix}
\lambda_D & \lambda_D \\
\lambda_D^* & \lambda_D^* \\
\end{vmatrix} = 
\begin{vmatrix}
\lambda_D & \lambda_D^* \\
\lambda_D & \lambda_D^* \\
\end{vmatrix}
= \lambda_D \lambda_D^* - \lambda_D \lambda_D^*
\]

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