Courts as Coordinators: Endogenous Enforcement and Jurisdiction in International Adjudication

Technical Appendix

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1 Fixed Costs (Exogenous Punishments) for Conflict

Suppose that there is a fixed cost for conflict, $f > 0$. This cost holds regardless of the size of endogenous punishments, c. We can interpret this as a punishment imposed by disinterested parties for reasons not captured in the game.

Bilateral bargaining disagreement payoffs are:

$$(d_i^B, d_j^B) = \begin{cases} (q(1-\phi)v_i - f,\ (1-q)(1-\phi)v_j - f) & \text{if } f < min\{q(1-\phi)v_i, (1-q)(1-\phi)v_j\} \\ (0, v_j) & \text{if } f \in [q(1-\phi)v_i, (1-q)(1-\phi)v_j] \\ (v_i, 0) & \text{if } f \in [(1-q)(1-\phi)v_j, q(1-\phi)v_i] \\ (0,0) & \text{if } f > max\{q(1-\phi)v_i, (1-q)(1-\phi)v_j\} \end{cases}$$

So the NBS for bilateral bargaining is:

$$x^B = \begin{cases} q(1-\phi) + \frac{\phi}{2} + \frac{f}{2v_j} - \frac{f}{2v_i} & \text{if } f < min\{q(1-\phi)v_i, (1-q)(1-\phi)v_j\} \\ 0 & \text{if } f \in [q(1-\phi)v_i, (1-q)(1-\phi)v_j] \\ 1 & \text{if } f \in [(1-q)(1-\phi)v_j, q(1-\phi)v_i] \\ \frac{1}{2} & \text{if } f > max\{q(1-\phi)v_i, (1-q)(1-\phi)v_j\} \end{cases}$$

<u>Case 1: Suppose $f < min\{q(1-\phi)v_i, (1-q)(1-\phi)v_j\}$</u>.

This yields the following disagreement payoffs for post-adjudicative bargaining if i wins:

$$(d_i^I, d_j^I) = \begin{cases} (w_i - k, w_j - k - c_{ij}) & \text{if } c_{ij} < (1-q)(1-\phi)v_j - f \\ (v_i - k, -k) & \text{if } c_{ij} \geq (1-q)(1-\phi)v_j - f \end{cases}$$

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If j wins, then the disagreement payoffs for post-adjudicative bargaining are:

$$(d_i^J, w_j^J) = \begin{cases} (w_i - k - c_{ji}, w_j - k) & \text{if } c_{ji} < q(1-\phi)v_i - f \\ (-k, v_j - k) & \text{if } c_{ji} \geq q(1-\phi)v_i - f \end{cases}$$

So the NBS for post-adjudicative bargaining is:

$$\begin{aligned} x^A(i) &= \begin{cases} x^B + \frac{c_{ij}}{2v_j} & \text{if } c_{ij} < (1-q)(1-\phi)v_j - f\text{; and} \\ 1 & \text{if } c_{ij} \geq (1-q)(1-\phi)v_j - f. \end{cases} \\ x^A(j) &= \begin{cases} x^B - \frac{c_{ji}}{2v_i} & \text{if } c_{ji} < q(1-\phi)v_i - f\text{; and} \\ 0 & \text{if } c_{ji} \geq q(1-\phi)v_i - f. \end{cases} \end{aligned}$$

All other results in the paper follow directly.

Case 2: Suppose $f \in [q(1-\phi)v_i, (1-q)(1-\phi)v_j]$.

Then post-adjudicative bargaining disagreement payoffs are:

$$(d_i, d_j) = \begin{cases} (v_i, 0) & \text{if } i \text{ wins and } c_{ij} \geq v_j - f\text{; and} \\ (0, v_j) & \text{if } i \text{ wins and } c_{ij} < v_j - f\text{; and} \\ (0, v_j) & \text{if } j \text{ wins.} \end{cases}$$

So the NBS for post-adjudicative bargaining is:

$$\begin{aligned} x^I &= \begin{cases} 1 & \text{if } c_{ij} \geq v_j - f\text{; and} \\ 0 & \text{if } c_{ij} < v_j - f. \end{cases} \\ x^J &= 0 \end{aligned}$$

Lemma 2 and parts (a) and (b) of Lemma 3 hold generally.

Part (c) of Lemma 3 only holds weakly. If $\rho = (1,0)$, then $c = 0$ when j wins. So $x^I(\rho = (1,0)) = x^I(\rho = (1,1))$ and $x^J(\rho = (1,0)) = x^J(\rho = (1,1))$ because x^J is invariant to c. So $I(1,1) = I(1,0)$. If $\rho = (0,1)$, then $c = 0$ when i wins. So $x^J(\rho = (0,1)) = x^J(\rho = (1,1))$ and $x^I(\rho = (0,1)) \leq x^I(\rho = (1,1))$. So $J(1,1) \subseteq J(0,1)$.

Using the proof technique for Lemma 4 in the paper and the results from the modified Lemma 3 above shows that if disputants have accepted jurisdiction: $V_n(\rho_n = 1) \geq V_n(\rho_n = 0)$.

Proposition 1 holds directly.

Note that:

$$px^I + (1-p)x^J - x^B = \begin{cases} p & \text{if } c_{ij} \geq v_j - f\text{; and} \\ 0 & \text{if } c_{ij} < v_j - f. \end{cases}$$

which is invariant to α-values. So part (a) of Lemma 5 no longer holds: submission decisions are not affected by α_i and α_j. This means that $\Delta_n(\rho_n = 1|\alpha_m)$ is neither increasing nor decreasing in α-values. So part (b) of Lemma 5 no longer holds: the expected benefit from the availability of the

Court is not affected by α_i and α_j.

Case 3: Suppose $f \in [(1-q)(1-\phi)v_j, q(1-\phi)v_i]$.

Then post-adjudicative bargaining disagreement payoffs are:

$$(d_i, d_j) = \begin{cases} (v_i, 0) & \text{if } i \text{ wins; and} \\ (0, v_j) & \text{if } j \text{ wins and } c_{ji} \geq v_i - f\text{; and} \\ (v_i, 0) & \text{if } j \text{ wins and } c_{ji} < v_i - f. \end{cases}$$

So the NBS for post-adjudicative bargaining is:

$$\begin{aligned} x^I &= 1 \\ x^J &= \begin{cases} 0 & \text{if } c_{ji} \geq v_i - f\text{; and} \\ 1 & \text{if } c_{ji} < v_i - f. \end{cases} \end{aligned}$$

Lemma 2 and parts (a) and (b) of Lemma 3 hold generally.

Part (c) of Lemma 3 only holds weakly. If $\rho = (1,0)$, then $c = 0$ when j wins. So $x^I(\rho = (1,0)) = x^I(\rho = (1,1))$ and $x^J(\rho = (1,0)) \geq x^J(\rho = (1,1))$. So $I(1,1) \subseteq I(1,0)$. If $\rho = (0,1)$, then $c = 0$ when i wins. So $x^J(\rho = (0,1)) = x^J(\rho = (1,1))$ and $x^I(\rho = (0,1)) = x^I(\rho = (1,1))$ because x^I is invariant to c. So $J(1,1) = J(0,1)$.

Using the proof technique for Lemma 3 in the paper and the results from the modified Proposition 1 above shows that: $V_n(\rho_n = 1) \geq V_n(\rho_n = 0)$.

Proposition 1 holds directly.

Note that:

$$px^I + (1-p)x^J - x^B = \begin{cases} p-1 & \text{if } c_{ji} \geq v_i - f\text{; and} \\ 0 & \text{if } c_{ji} < v_i - f. \end{cases}$$

which is invariant to α-values. So part (a) of Lemma 5 no longer holds: submission decisions are not affected by α_i and α_j. This means that $\Delta_n(\rho_n = 1|\alpha_m)$ is neither increasing nor decreasing in α-values. So part (b) of Lemma 5 no longer holds: the expected benefit from the availability of the Court is not affected by α_i and α_j.

Case 4: Suppose $f > max\{q(1-\phi)v_i, (1-q)(1-\phi)v_j\}$.

Then post-adjudicative bargaining disagreement payoffs are: $(d_i^I, d_j^I) = (v_i, 0)$ and $(d_i^J, d_j^J) = (0, v_j)$. So the NBS for post-adjudicative bargaining is: $x^I = 1$ and $x^J = 0$.

Lemma 2 and parts (a) and (b) of Lemma 3 hold generally.

Part (c) of Lemma 3 only holds weakly. Since both x^I and x^J are invariant to c, $I(1,1) = I(1,0)$ and $J(1,1) = J(0,1)$.

Using the proof technique for Lemma 4 in the paper and the results from the modified Lemma 3 above shows that: $V_n(\rho_n = 1) = V_n(\rho_n = 0)$.

Derivations in the Proof of Proposition 1 hold directly.

Note that:

$$px^I + (1-p)x^J - x^B = p - \frac{1}{2}$$

which is invariant to α-values. So part (a) of Lemma 5 no longer holds: submission decisions are not affected by α_i and α_j. This means that $\Delta_n(\rho_n = 1|\alpha_m)$ is neither increasing nor decreasing in α-values. So part (b) of Lemma 5 no longer holds: the expected benefit from the availability of the Court is not affected by α_i and α_j.

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Note that which of the four cases holds depends upon the draw of player types, (v_i, v_j). As long as there is positive probability that $f < min\{q(1-\phi)v_i, (1-q)(1-\phi)v_j\}$, then all of the causal mechanisms identified in the model hold because all of the relevant relations (e.g. $V_n(\rho_n = 1) > V_n(\rho_n = 0)$, $\Delta_n(\rho_n = 1|\alpha_m)$ decreasing in α_n and increasing in α_m in an HOS, etc.) hold in expectation.

2 Alternative Bargaining Solutions

Let $x(w_i, w_j)$ denote i's equilibrium share from the induced bargaining games. Suppose:

$$\frac{\partial x}{\partial w_i} > 0 \text{ and } \frac{\partial x}{\partial w_j} < 0$$

Lemmata 2-4 and Proposition 1 still hold.

To establish part (a) of Lemma 5, recall the case submission IC constraints: $\left[px^I + (1-p)x^J - x^B\right] v_i - k \geq 0$ and $\left[x^B - px^I - (1-p)x^J\right] v_j - k \geq 0$. Then differentiating with respect to i's constraint yields:

$$\begin{aligned}
\frac{\partial}{\partial \alpha_i}\left[px^I + (1-p)x^J - x^B\right] &= \frac{\partial x}{\partial w_i}\left[p\frac{\partial}{\partial \alpha_i}w_i^I + (1-p)\frac{\partial}{\partial \alpha_i}w_i^J - \frac{\partial}{\partial \alpha_i}w_i^B\right] \\
\frac{\partial}{\partial \alpha_j}\left[px^I + (1-p)x^J - x^B\right] &= \frac{\partial x}{\partial w_j}\left[p\frac{\partial}{\partial \alpha_j}w_j^I + (1-p)\frac{\partial}{\partial \alpha_j}w_j^J - \frac{\partial}{\partial \alpha_j}w_j^B\right]
\end{aligned}$$

In an HOS:

$$\begin{array}{lll}
\frac{\partial}{\partial \alpha_i}w_i^I = 0 & \frac{\partial}{\partial \alpha_i}w_i^J = 0 & \frac{\partial}{\partial \alpha_i}w_i^B = \frac{\partial q}{\partial \alpha_i}(1-\phi)v_i > 0 \\
\frac{\partial}{\partial \alpha_j}w_j^I = 0 & \frac{\partial}{\partial \alpha_j}w_j^J = 0 & \frac{\partial}{\partial \alpha_j}w_j^B = -\frac{\partial q}{\partial \alpha_j}(1-\phi)v_j > 0
\end{array}$$

So:

$$\frac{\partial}{\partial \alpha_i}\left[px^I + (1-p)x^J - x^B\right] = -\frac{\partial x}{\partial w_i}\frac{\partial w_i^B}{\partial \alpha_i} < 0 \tag{1}$$

$$\frac{\partial}{\partial \alpha_j}\left[px^I + (1-p)x^J - x^B\right] = -\frac{\partial x}{\partial w_j}\frac{\partial w_j^B}{\partial \alpha_j} > 0 \tag{2}$$

It also follows that case submission constraints are equivalent to:

$$\frac{\partial}{\partial \alpha_i}\left[x^B - px^I - (1-p)x^J\right] > 0$$
$$\frac{\partial}{\partial \alpha_j}\left[x^B - px^I - (1-p)x^J\right] < 0$$

So $I(\rho)$ contracts in α_i and expands in α_j, while $J(\rho)$ expands in α_i and contracts in α_j.

To see that part (b) of Lemma 5 still holds, note that on the equilibrium path, states that accept jurisdiction are always providers of enforcement. So:

$$\begin{aligned}\Delta_n(\rho_n = 1|\alpha_m) &= \frac{1}{2}\Bigg[\iint\limits_{I(1,1|i=n)\cup J(1,1|i=n)} \left[\left(px^I_{i=n} + (1-p)x^J_{i=n} - x^B_{i=n}\right)v_i - k\right] dF(v_i)dF(v_j) \\ &+ \iint\limits_{I(1,1|i=m)\cup J(1,1|i=m)} \left[\left(x^B_{i=m} - px^I_{i=m} - (1-p)x^J_{i=m}\right)v_j - k\right] dF(v_i)dF(v_j)\Bigg]\end{aligned}$$

Suppose $i = n$. Then an increase in α_n decreases the integrand for all points, while contracting $I(1,1|i=n)$ and expanding $J(1,1|i=n)$. Similarly, an increase in α_m increases the integrand for all points, while expanding $I(1,1|i=n)$ and contracting $J(1,1|i=n)$. Analogous reasoning holds for $i = m$.

Proposition 3 follows if $x(w_i = w_j) = \frac{1}{2}$. All other results follow directly.

3 Court Rulings as a Function of Asset Valuations

Let $p(v_i, v_j)$ be the probability that player i wins the Court ruling, given pair (v_i, v_j). Suppose:

- p is continuously differentiable in both of its arguments
- $p_1 > 0$ and $p_2 < 0$

Note that all results excepting Proposition 3 hold directly based on proofs in the original paper.

To demonstrate existence, choose $\alpha_n > \alpha_m$, per the original strategy of the Proof of Proposition 3. Then: $x^B_{i=m} = 1 - x^B_{i=n} < \frac{1}{2} < x^B_{i=n}$.

Suppose:

$$p(v_i, v_j) = x^B_{i=n} + \epsilon(v_i, v_j)$$

s.t. $\epsilon(v_i, v_j) > 0$ for all (v_i, v_j) pairs, ϵ is continuously differentiable in both of its arguments, $\epsilon_1 > 0$, $\epsilon_2 < 0$, and:

$$\epsilon(1,0) \in \left(2k, 1 - x^B_{i=n}\right)$$

Then p is well-defined. [Note that we can always choose k s.t. $2k < 1 - x^B_{i=n}$.]

Note that $x^B_{i=m} < x^B_{i=n} < p(v_i, v_j)$ always, so $J(1,1|i=n) = J(1,1|i=m) = \emptyset$. Also:

$$\begin{aligned} I(1,1|i=n) &= \left\{(v_i, v_j) \mid (p - x^B_{i=n})v_i \geq k\right\} = \{(v_i, v_j) \mid \epsilon(v_i, v_j)v_i \geq k\} \\ I(1,1|i=m) &= \left\{(v_i, v_j) \mid (p - x^B_{i=m})v_i \geq k\right\} \\ &= \{(v_i, v_j) \mid \left[\epsilon(v_i, v_j) + (1-\phi)(2q_{nm} - 1)\right] v_i \geq k\} \end{aligned}$$

Note that $\epsilon(1,0) > 2k > k$ and $q_{nm} > \frac{1}{2}$ ensure that each player is sometimes willing to submit a case when it is assigned to role i.

The benefit to player n of having accepted jurisdiction of the Court when he is matched against player m is:

$$\begin{aligned} \Delta_n(\rho_n = 1|\alpha_m) &= \frac{1}{2}\left[\iint_{I(1,1|i=n)} \left[(p - x^B_{i=n})v_i - k\right] dF(v_i)dF(v_j)\right. \\ &\qquad \left. + \iint_{I(1,1|i=m)} \left[(1 - x^B_{i=n} - p)v_j - k\right] dF(v_i)dF(v_j)\right] \\ &\propto \iint_{I(1,1|i=n)} \left[\epsilon(v_i, v_j)v_i - k\right] dF(v_i)dF(v_j) \\ &\qquad + \iint_{I(1,1|i=m)} \{\left[(1-\phi)(1 - 2q_{nm}) - \epsilon(v_i, v_j)\right] v_j - k\} \, dF(v_i)dF(v_j) \end{aligned}$$

Note that:

$$\lim_{\alpha_n \to \alpha_m} I(1,1|i=m) = I(1,1|i=n)$$

So:

$$\lim_{\alpha_n \to \alpha_m} \Delta_n(\rho_n = 1|\alpha_m) \propto \iint_{\{(v_i, v_j) \mid \epsilon(v_i, v_j)v_i \geq k\}} \left[\epsilon(v_i, v_j)(v_i - v_j) - 2k\right] dF(v_i)dF(v_j)$$

Consider the condition in the bounds of integration: $\epsilon(v_i, v_j)v_i \geq k$.

Define:

$$\underline{v}_i \equiv \min_{v_i \in [0,1]} \{\epsilon(v_i, 0)v_i = k\}$$

[Note that $\epsilon(1, 0) > 2k > k$ ensures that $\underline{v}_i < 1$.] Then $\epsilon(v_i, 0)v_i > k$ for all $v_i > \underline{v}_i$.

For any such $v_i > \underline{v}_i$, define $\hat{v}_j(v_i)$ as the value of v_j s.t. $\epsilon(v_i, \hat{v}_j(v_i))v_i = k$.

Then $\epsilon(v_i, \hat{v}_j(v_i))v_i > k$ for all $v_j < \hat{v}_j(v_i)$.

So:

$$\lim_{\alpha_n \to \alpha_m} \Delta_n(\rho_n = 1|\alpha_m) \quad \propto \quad \int_{\underline{v}_i}^{1} \int_{0}^{\hat{v}_j(v_i)} \left[\epsilon(v_i, v_j)(v_i - v_j) - 2k\right] dF(v) dF(v)$$

Note that the integrand is strictly positive iff: $\epsilon(v_i, v_j)(v_i - v_j) > 2k$. The LHS reaches its maximum value when $v_i = 1$ and $v_j = 0$. Recall that $\epsilon(1, 0) > 2k$. So the integrand is strictly positive for valuation pairs (v_i, v_j) in which there are relatively high values of v_i and relatively low values of v_j. Note that when v_i is low and v_j is high, cases are not submitted. So if there is sufficiently high density on the extreme regions of the unit interval, then $\lim_{\alpha_n \to \alpha_m} \Delta_n(\rho_n = 1|\alpha_m) > 0$.

Note that the second part of the proof of Proposition 3—showing that an HOS can be supported by equilibrium enforcement behavior—continues to hold without alteration.

4 Divisibility by the Court

Suppose that the Court is able to issue a ruling that consists of an allocation for player i, $\pi \in [0, 1]$. So the asset is fully divisible by the Court and player j receives a share $1 - \pi$. Following such a ruling, we will treat the allocation as consisting of two distinct prizes.

Let $c_{ij\pi}$ denote the level of enforcement provided for conflict over i's share of the asset when the Court allocates share π to player i in his dispute against player j. Let $c_{ji\pi}$ denote the level of enforcement provided for conflict over j's share of the asset when the Court allocates share π to player i in his dispute against player j.

Note that player i is willing to fight for player j's share of the asset iff: $q(1 - \phi)v_i(1 - \pi) > c_{ji\pi}$. Similarly, player j is willing to fight for player i's share of the asset iff: $(1 - q)(1 - \phi)v_j\pi > c_{ij\pi}$.

The post-adjudicative bargaining game over j's prize has the following disagreement payoffs:

$$d_i(j \text{ prize}) = \begin{cases} q(1-\phi)v_i(1-\pi) - k - c_{ji\pi} & \text{if } q(1-\phi)v_i(1-\pi) > c_{ji\pi}; \\ -k & \text{if } q(1-\phi)v_i(1-\pi) \leq c_{ji\pi}. \end{cases}$$

$$d_j(j \text{ prize}) = \begin{cases} (1-q)(1-\phi)v_j(1-\pi) - k & \text{if } q(1-\phi)v_i(1-\pi) > c_{ji\pi}; \\ v_j(1-\pi) - k & \text{if } q(1-\phi)v_i(1-\pi) \leq c_{ji\pi}. \end{cases}$$

This yields the following NBS over j's prize:

$$x^A(j \text{ prize}) = \begin{cases} x^B - \frac{c_{ji\pi}}{2v_i(1-\pi)} & \text{if } q(1-\phi)v_i(1-\pi) > c_{ji\pi}; \\ 0 & \text{if } q(1-\phi)v_i(1-\pi) \leq c_{ji\pi}. \end{cases}$$

The post-adjudicative bargaining game over i's prize has the following disagreement payoffs:

$$d_i(i \text{ prize}) = \begin{cases} q(1-\phi)v_i\pi - k & \text{if } (1-q)(1-\phi)v_j\pi > c_{ij\pi}; \\ v_i\pi - k & \text{if } (1-q)(1-\phi)v_j\pi \leq c_{ij\pi}. \end{cases}$$

$$d_j(i \text{ prize}) = \begin{cases} (1-q)(1-\phi)v_j\pi - k - c_{ij\pi} & \text{if } (1-q)(1-\phi)v_j\pi > c_{ij\pi}; \\ -k & \text{if } (1-q)(1-\phi)v_j\pi \leq c_{ij\pi}. \end{cases}$$

This yields the following NBS over i's prize:

$$x^A(i \text{ prize}) = \begin{cases} x^B + \frac{c_{ij\pi}}{2v_j\pi} & \text{if } (1-q)(1-\phi)v_j\pi > c_{ij\pi}; \\ 1 & \text{if } (1-q)(1-\phi)v_j\pi \leq c_{ij\pi}. \end{cases}$$

So the overall NBS for post-adjudicative bargaining over both prizes is:

$$\begin{aligned} x^A(\pi) &= \pi x^A(i \text{ prize}) + (1-\pi)x^A(j \text{ prize}) \\ &= \begin{cases} x^B + \frac{c_{ij\pi}}{2v_j} - \frac{c_{ji\pi}}{2v_i} & \text{if } q(1-\phi)v_i(1-\pi) > c_{ji\pi} \\ & \quad \text{and } (1-q)(1-\phi)v_j\pi > c_{ij\pi} \\ \pi x^B + \frac{c_{ij\pi}}{2v_j} & \text{if } q(1-\phi)v_i(1-\pi) \leq c_{ji\pi} \\ & \quad \text{and } (1-q)(1-\phi)v_j\pi > c_{ij\pi} \\ x^B + \pi(1-x^B) - \frac{c_{ji\pi}}{2v_i} & \text{if } q(1-\phi)v_i(1-\pi) > c_{ji\pi} \\ & \quad \text{and } (1-q)(1-\phi)v_j\pi \leq c_{ij\pi} \\ \pi & \text{if } q(1-\phi)v_i(1-\pi) \leq c_{ji\pi} \\ & \quad \text{and } (1-q)(1-\phi)v_j\pi \leq c_{ij\pi} \end{cases} \end{aligned}$$

Lemma 2 holds directly.

Suppose that the Court chooses $\pi \in [0,1]$ according to the density function $f(\cdot)$.

Then player i's expected utility from adjudication is:

$$\int_0^1 [x^A(\pi)v_i - k]f(\pi)d\pi = v_i \int_0^1 x^A(\pi)f(\pi)d\pi - k$$

Similarly, player j's expected utility from adjudication is:

$$\int_0^1 \left\{ \left[1 - x^A(\pi)\right] v_j - k \right\} f(\pi) d\pi = v_j \left[1 - \int_0^1 x^A(\pi) f(\pi) d\pi \right] - k$$

So the incentive compatibility constraints for case submission are:

$$v_i \int_0^1 x^A(\pi) f(\pi) d\pi - k \geq x^B v_i \quad \Leftrightarrow \quad v_i \left[\int_0^1 x^A(\pi) f(\pi) d\pi - x^B\right] - k \geq 0 \tag{3}$$

$$v_j \left[1 - \int_0^1 x^A(\pi) f(\pi) d\pi\right] - k \geq (1 - x^B) v_j \quad \Leftrightarrow \quad v_j \left[x^B - \int_0^1 x^A(\pi) f(\pi) d\pi\right] - k \geq 0 \tag{4}$$

Both constraints can't hold simultaneously, which establishes part (a) of Lemma 3.

If player i is a free-rider, then $c_{ij\pi} = 0$ when j fights for i's prize. This means:

$$x^A(\pi) = \begin{cases} x^B - \frac{c_{ji\pi}}{2v_i} & \text{if } c_{ji\pi} < q(1-\phi)v_i(1-\pi) \text{ [so } i \text{ is willing to fight over } j\text{'s prize]} \\ \pi x^B & \text{if } c_{ji\pi} \geq q(1-\phi)v_i(1-\pi) \text{ [so } i \text{ is not willing to fight over } j\text{'s prize]} \end{cases}$$

This means that (3) never holds, so player i is never willing to submit a case to the Court.

If player j is a free-rider, then $c_{ji\pi} = 0$ when i fights for j's prize. This means:

$$x^A(\pi) = \begin{cases} x^B + \frac{c_{ij\pi}}{2v_j} & \text{if } c_{ij\pi} < (1-q)(1-\phi)v_j\pi \text{ [so } j \text{ is willing to fight for } i\text{'s prize]} \\ x^B + \pi(1 - x^B) & \text{if } c_{ij\pi} \geq (1-q)(1-\phi)v_j\pi \text{ [so } j \text{ is not willing to fight for } i\text{'s prize]} \end{cases}$$

This means that (4) never holds, so player j is never willing to submit a case to the Court.

So free riders will never submit cases to the Court, which establishes part (b) of Lemma 3.

Part (c) of Lemma 3 follows directly.

Choose an arbitrary (n, m) who have accepted jurisdiction. Suppose that the probability of trial is positive. Then:

$$\begin{aligned} V_n(\rho|\alpha_m) &= Pr(i=n)V_n(\rho|i=n) + Pr(i=m)V_n(\rho|i=m) \\ &= \frac{1}{2}\Bigg\{ \iint\limits_{I(\rho|i=n)\cup J(\rho|i=n)} \left[\left(\int_0^1 x^A_{i=n}(\pi|v_i,v_j)f(\pi)d\pi - x^B_{i=n}\right) v_i - k\right] dF(v_i)dF(v_j) \\ &\quad + \iint\limits_{I(\rho|i=m)\cup J(\rho|i=m)} \left[\left(x^B_{i=m} - \int_0^1 x^A_{i=m}(\pi|v_i,v_j)f(\pi)d\pi\right) v_j - k\right] dF(v_i)dF(v_j) \\ &\quad + \iint\limits_{[0,1]^2} x^B_{i=n} v_i dF(v_i)dF(v_j) + \iint\limits_{[0,1]^2} (1 - x^B_{i=m}) v_j dF(v_i)dF(v_j) \Bigg\} \end{aligned} \tag{5}$$

By the Proof of Lemma 3, expansions in $I(\rho)$ raise i's utility and decrease j's utility; the opposite holds for $J(\rho)$. Then parts (b) and (c) of Lemma 3 establish Lemma 4.

Proposition 1 follows directly.

Adopt the definition of an HOS from the text of the paper. Then $x^A(\pi) = \pi$ always, and:

$$E[x^A(\pi)] = \int_0^1 \pi f(\pi) d\pi = E[\pi]$$

So:

$$\begin{array}{rcll} (\dagger) & = & (E[\pi] - x^B)v_i - k & \Leftrightarrow \quad (3) \text{ holds} \\ (\ddagger) & = & (x^B - E[\pi])v_j - k & \Leftrightarrow \quad (4) \text{ holds} \end{array}$$

Note that $\frac{\partial}{\partial \alpha_i}(\dagger) < 0$, $\frac{\partial}{\partial \alpha_j}(\dagger) > 0$, $\frac{\partial}{\partial \alpha_i}(\ddagger) > 0$, and $\frac{\partial}{\partial \alpha_j}(\ddagger) < 0$. So part (a) of Lemma 5 is established.

On the equilibrium path, states that accept jurisdiction are always providers. So by eqn (5), in an HOS:

$$\begin{aligned} \Delta_n(\rho_n = 1|\alpha_m) = \frac{1}{2} \Bigg[& \iint_{I(1,1|i=n) \cup J(1,1|i=n)} \left[(E[\pi] - x^B_{i=n})v_i - k\right] dF(v_i)dF(v_j) \\ + & \iint_{I(1,1|i=m) \cup J(1,1|i=m)} \left[(x^B_{i=m} - E[\pi])v_j - k\right] dF(v_i)dF(v_j) \Bigg] \end{aligned}$$

Suppose $i = n$. Then an increase in α_n contracts $I(1,1|i = n)$ and expands $J(1,1|i = n)$ by (a). Similarly, an increase in α_m expands $I(1,1|i = n)$ and contracts $J(1,1|i = n)$. Additionally, (a) shows that the first integrand is decreasing in α_n and increasing in α_m at all points. So the first term of $\Delta_n(1|\alpha_m)$ is decreasing in α_n and increasing in α_m. Suppose $i = m$ and apply the same proof strategy to the second term of $\Delta_n(1|\alpha_m)$. Then part (b) of Lemma 5 holds.

To establish that Proposition 3 still holds, note that $E[\pi]$ is a parameter just like p. So the same proof strategy works in this extension.

All other model results follow directly.

5 Outside Options

Suppose that adjudication serves as an outside option when disputants have accepted jurisdiction.

All behavior in the adjudication subgame is unaffected, so Lemma 1 still holds for post-adjudicative bargaining. Lemma 2 and parts (a) and (b) of Lemma 3 also continue to hold without alteration.

The outside options for bilateral bargaining are defined by:

$$\begin{aligned}\gamma_i &= \left[px^I + (1-p)x^J\right] v_i - k \\ \gamma_j &= \left[p\left(1-x^I\right) + (1-p)\left(1-x^J\right)\right] v_j - k\end{aligned}$$

So:

$$\begin{aligned}xv_i &= \gamma_i = \left[px^I + (1-p)x^J\right] v_i - k \\ \Leftrightarrow x &= px^I + (1-p)x^J - \frac{k}{v_i} \equiv p_i\end{aligned}$$

and:

$$\begin{aligned}(1-x)v_j &= \gamma_j = \left[p\left(1-x^I\right) + (1-p)\left(1-x^J\right)\right] v_j - k \\ \Leftrightarrow x &= 1 - \left[p\left(1-x^I\right) + (1-p)\left(1-x^J\right) - \frac{k}{v_j}\right] \\ &= px^I + (1-p)x^J + \frac{k}{v_j} \equiv p_j\end{aligned}$$

So $p_i < p_j$ always and the constrained bilateral bargaining interval exists: $X^C \equiv [p_i, p_j]$.

So the constrained NBS for bilateral bargaining is:

$$x^C = \begin{cases} p_i & \text{if } x^B \in [0, p_i)\text{; and} \\ x^B & \text{if } x^B \in [p_i, p_j]\text{; and} \\ p_j & \text{if } x^B \in (p_j, 1]. \end{cases}$$

This means that cases will never be submitted in equilibrium because x^C induces indifference anytime $x^B \in [0, p_i)$ or $x^B \in (p_j, 1]$.

Now:

$$\begin{aligned}I^C(\rho) &\equiv \{(v_i, v_j) | v_i, v_j \in [0,1] \ \wedge \ x^B \in [0, p_i)\} = I(\rho) \\ J^C(\rho) &\equiv \{(v_i, v_j) | v_i, v_j \in [0,1] \ \wedge \ x^B \in (p_j, 1]\} = J(\rho)\end{aligned}$$

So the analogue of part (c) of Lemma 3 follows directly: $I^C(1,1) \subset I^C(1,0)$ and $J^C(1,1) \subset J^C(0,1)$.

To establish Lemma 4, note that $\neg(I(\rho) = \emptyset \wedge J(\rho) = \emptyset) \ \Leftrightarrow \ \neg(I^C(\rho) = \emptyset \wedge J^C(\rho) = \emptyset)$ and choose an arbitrary (n,m) who have accepted jurisdiction. Then:

$$\begin{aligned}
V_n(\rho|\alpha_m) &= Pr(i=n)V_n(\rho|i=n) + Pr(i=m)V_n(\rho|i=m) \\
&= \frac{1}{2}\Bigg[\iint\limits_{I^C(\rho|i=n)} \left(p_i - x^B_{i=n}\right) v_i dF(v_i)dF(v_j) + \iint\limits_{J^C(\rho|i=n)} \left(p_j - x^B_{i=n}\right) v_i dF(v_i)dF(v_j) \\
&\quad + \iint\limits_{I^C(\rho|i=m)} \left(x^B_{i=m} - p_i\right) v_j dF(v_i)dF(v_j) + \iint\limits_{J^C(\rho|i=m)} \left(x^B_{i=m} - p_j\right) v_j dF(v_i)dF(v_j) \\
&\quad + \iint\limits_{[0,1]^2} x^B_{i=n} v_i dF(v_i)dF(v_j) + \iint\limits_{[0,1]^2} \left(1 - x^B_{i=m}\right) v_j dF(v_i)dF(v_j)\Bigg]
\end{aligned} \tag{6}$$

By the Proof of Lemma 3, expansions in $I^C(\rho)$ raise i's utility and decrease j's utility; the opposite holds for $J^C(\rho)$. Then parts (b) and (c) of Lemma 3 establish Lemma 4.

Note that Proposition 1 continues to hold.

The analogue of part (a) of Lemma 5 follows directly: $I^C(\rho)$ is expanding in α_j and contracting in α_i, while $J^C(\rho)$ is expanding in α_i and contracting in α_j.

To establish part (b) of Lemma 5, note that on the equilibrium path, states that accept jurisdiction are always providers. So by eqn (6):

$$\begin{aligned}
\Delta_n(\rho_n = 1|\alpha_m) &= \frac{1}{2}\Bigg[\iint\limits_{I^C(1,1|i=n)} \left(p_i - x^B_{i=n}\right) v_i dF(v_i)dF(v_j) + \iint\limits_{J^C(1,1|i=n)} \left(p_j - x^B_{i=n}\right) v_i dF(v_i)dF(v_j) \\
&\quad + \iint\limits_{I^C(1,1|i=m)} \left(x^B_{i=m} - p_i\right) v_j dF(v_i)dF(v_j) + \iint\limits_{J^C(1,1|i=m)} \left(x^B_{i=m} - p_j\right) v_j dF(v_i)dF(v_j)\Bigg]
\end{aligned}$$

Suppose $i = n$. Then an increase in α_n contracts $I^C(1,1|i=n)$ and expands $J^C(1,1|i=n)$ by the analogue of part (c) of Lemma 3 above. Similarly, an increase in α_m expands $I^C(1,1|i=n)$ and contracts $J^C(1,1|i=n)$. By definition, $p_i - x^B > 0$ for all pairs in $I^C(\rho)$ and $x^B - p_j > 0$ for all pairs in $J^C(\rho)$. Also, $p_i - x^B \propto px^I + (1-p)x^J - x^B$ and $x^B - p_j \propto x^B - px^I - (1-p)x^J$. So the original proof of part (b) of Lemma 5 holds.

To establish Proposition 3, note that in an HOS if $\rho = (1,1)$:

$$p_i = p - \frac{k}{v_i} \quad \text{and} \quad p_j = p + \frac{k}{v_j}$$

If $p \equiv x^B_{i=n} + \epsilon$, then a state will never want to submit a case to the Court when it is assigned to role j (i.e. $J(1,1|i=n) = J(1,1|i=m) = \emptyset$). So:

$$
\begin{aligned}
\Delta_n(\rho_n = 1|\alpha_m) \;\; &\propto \;\; \iint\limits_{I^C(1,1|i=n)} \left[\epsilon v_i - k\right] dF(v_i)dF(v_j) \\
&\quad + \iint\limits_{I^C(1,1|i=m)} \left\{\left[(1-\phi)(1-2q_{nm}) - \epsilon\right] v_j + \frac{kv_j}{v_i}\right\} dF(v_i)dF(v_j)
\end{aligned}
$$

where:

$$
\begin{aligned}
I^C(1,1|i=n) \;\; &= \;\; \left\{(v_i,v_j)|x^B_{i=n} < p - \frac{k}{v_i} \Leftrightarrow v_i > \frac{k}{p - x^B_{i=n}} = \frac{k}{\epsilon} \equiv \hat{v}_{in}\right\} \\
I^C(1,1|i=m) \;\; &= \;\; \left\{(v_i,v_j)|x^B_{i=m} < p - \frac{k}{v_i} \Leftrightarrow v_i > \frac{k}{p - (1 - x^B_{i=n})} = \frac{k}{\epsilon + (1-\phi)(2q_{nm}-1)} \equiv \hat{v}_{im}\right\}
\end{aligned}
$$

So:

$$
\begin{aligned}
\Delta_n(1|\alpha_m) \;\; &\propto \;\; \int_{\hat{v}_{in}}^{1} \left[\epsilon v - k\right] dF(v) + \int_{\hat{v}_{im}}^{1} \left\{\left[(1-\phi)(1-2q_{nm}) - \epsilon\right] E[v] + \frac{kE[v]}{v}\right\} dF(v) \\
\Rightarrow \lim_{\alpha_n \to \alpha_m} \Delta_n(1|\alpha_m) \;\; &\propto \;\; \int_{\frac{k}{\epsilon}}^{1} \left[\epsilon(v - E[v]) - k + \frac{kE[v]}{v}\right] dF(v)
\end{aligned}
$$

By the argument in the Proof of Proposition 3, there exist parameters for which this is positive.

Recall that in order to show that an HOS can be supported, it is sufficient to demonstrate that $\lim_{\alpha \to \bar{\alpha}} \Delta_n(\rho_n = 0|\alpha_m) < 0$.

If player n is a free-rider ($\rho_n = 0$), then $I^C(\rho|i=n) = J^C(\rho|i=m) = \emptyset$. Also:

$$
p_i(i=m) = p + (1-p)x^B_{i=m} - \frac{k}{v_i} \quad \text{and} \quad p_j(i=n) = px^B_{i=n} + \frac{k}{v_j}
$$

So:

$$\begin{aligned}
\Delta_n(\rho_n = 0|\alpha_m) \quad \propto & \quad \iint\limits_{J^C(0,1|i=n)} \left(p_j - x^B_{i=n}\right) v_i dF(v_i)dF(v_j) + \iint\limits_{I^C(1,0|i=m)} \left(x^B_{i=m} - p_i\right) v_j dF(v_i)dF(v_j) \\
= & \quad \iint\limits_{J^C(0,1|i=n)} \left[(p-1)x^B_{i=n}v_i + \frac{kv_i}{v_j}\right] dF(v_i)dF(v_j) \\
& \quad + \iint\limits_{I^C(1,0|i=m)} \left[-px^B_{i=n}v_j + \frac{kv_j}{v_i}\right] dF(v_i)dF(v_j)
\end{aligned}$$

where:

$$\begin{aligned}
J^C(0,1|i=n) &= \left\{(v_i, v_j)|px^B_{i=n} + \frac{k}{v_j} < x^B_{i=n} \Leftrightarrow v_j \geq \frac{k}{(1 - x^B_{i=n} - \epsilon)x^B_{i=n}}\right\} \\
\Rightarrow \lim_{\alpha\to\bar{\alpha}} J^C(0,1|i=n) &= \left\{(v_i, v_j) \mid v_j \geq \frac{4k}{1-2\epsilon}\right\} \\
I^C(1,0|i=m) &= \left\{(v_i, v_j)|x^B_{i=m} < p + (1-p)x^B_{i=m} - \frac{k}{v_i} \Leftrightarrow v_i \geq \frac{k}{(x^B_{i=n} + \epsilon)x^B_{i=n}}\right\} \\
\Rightarrow \lim_{\alpha\to\bar{\alpha}} I^C(1,0|i=m) &= \left\{(v_i, v_j) \mid v_i \geq \frac{4k}{1+2\epsilon}\right\}
\end{aligned}$$

So:

$$\begin{aligned}
\lim_{\alpha\to\bar{\alpha}} \Delta_n(\rho_n = 0|\alpha_m) \quad \propto & \quad \int\limits_{\frac{4k}{1-2\epsilon}}^{1} \lim_{\alpha\to\bar{\alpha}} \left[E[v]\left(\frac{k}{v} - (1-p)x^B_{i=n}\right)\right] dF(v) \\
& \quad + \int\limits_{\frac{4k}{1+2\epsilon}}^{1} \lim_{\alpha\to\bar{\alpha}} \left[E[v]\left(\frac{k}{v} - px^B_{i=n}\right)\right] dF(v) \\
= & \quad \int\limits_{\frac{4k}{1-2\epsilon}}^{1} E[v]\left(\frac{k}{v} - \frac{1-2\epsilon}{4}\right) dF(v) \\
& \quad + \int\limits_{\frac{4k}{1+2\epsilon}}^{1} E[v]\left(\frac{k}{v} - \frac{1+2\epsilon}{4}\right) dF(v) < 0
\end{aligned}$$

because the integrands are negative for the support of their respective integrals. So Proposition 3 holds in this model extension.

All of the other results of the model follow directly.

6 Finite Reputational Loss

Suppose that free-riding on enforcement costs in a particular period results in a 'bad' reputation for a finite number X of periods. Then for each player $n \in N$ and history of action h^t, let $\rho_n(h^t) = 0$ if there exists a past period $t' \in [t-X, t-1]$ in which n was a disinterested player, the winner of adjudication was a provider, and n provided enforcement $e_n < \hat{e}_n$. Otherwise, let $\rho_n(h^t) = 1$.

Lemma 1 continues to hold and the following parts of Lemma 2 follow directly from the proof in the paper: $e_n \in \{0, \hat{e}_n\}$ in equilibrium; $e_n = 0$ if the winner of adjudication is a free-rider; and $e_n = 0$ if n has refused jurisdiction of the court. [The claim that $e_n = 0$ if n has a reputation as a free-rider at an enforcement decision-node does not follow directly from the original proof of Lemma 1. However, this will be an implication of the revised proof of Proposition 1 below.]

Lemmata 3 and 4 follow directly from the proofs in the paper.

To establish the analogue of Proposition 1, consider a player n who has accepted jurisdiction of the court and reaches a decision-node at which she must decide whether to enforce a ruling in which the winner is a provider of enforcement. In equilibrium, the probability of arriving at such a decision-node is zero. If player n adopts a strategy in which she always pays the enforcement threshold cost when the winner of the dispute is a provider and does not enforce otherwise, then her expected utility at this decision-node is: $-\hat{e}_n + \sum_{t=t'+1}^{\infty} \delta^{t-t'} \frac{2}{|N|} W_n(\rho_n = 1)$.[1] Player n's expected utility from providing zero enforcement at this decision-node is:

$$\begin{aligned} & \left[\delta + \delta^2 + ... + \delta^X\right] \left\{ \frac{2}{|N|} W_n(\rho_n = 0) \right\} + \sum_{t=t'+X+1}^{\infty} \delta^{t-t'} \left\{ \frac{2}{|N|} W_n(\rho_n = 1) \right\} \\ = \quad & \frac{\delta}{1-\delta} \left\{ \frac{2}{|N|} W_n(\rho_n = 1) \right\} + \frac{\delta\left(1-\delta^X\right)}{1-\delta} \left\{ \frac{2}{|N|} [W_n(\rho_n = 0) - W_n(\rho_n = 1)] \right\} \end{aligned}$$

So player n will enforce if and only if:

$$\hat{e}_n \leq \frac{\delta\left(1-\delta^X\right)}{1-\delta} \frac{2}{|N|} E[V_n(\rho_n = 1) - V_n(\rho_n = 0)]$$

To see that Proposition 3 still holds, note that by l'Hopital's rule:

$$\lim_{\delta \to 1} \frac{\delta\left(1-\delta^X\right)}{1-\delta} = \lim_{\delta \to 1} \left[\delta^X(1+X) - 1\right] = X$$

So the argument in the Proof of Proposition 3 follows directly when X is treated as an exogenous parameter of the game. All of the other results of the model follow directly.

[1] Note that this dominates a strategy in which n enforces at this node, but not necessarily at other nodes. This strategy yields an expected utility of $-\hat{e}_n + \sum_{t=t'+1}^{\infty} \delta^{t-t'} \frac{2}{|N|} W_n(\rho_n(h^t))$.

7 Enforcement for Free-Riders

In the model presented in the paper, non-disputants are permitted to provide zero enforcement for free-riders without injuring their own reputation. Are results robust to changing this assumption?

7.1 Equal Enforcement for Free-Riders

Suppose that in order to maintain her reputation as a provider, every non-disputant is expected to provide a level of enforcement, $\hat{e}_n$, that does not vary with respect to the enforcement parameter of the winner of adjudication. Then the equilibrium level of enforcement for a particular court ruling is invariant to ρ. Part (a) of Lemma 2 no longer holds, but parts (b) and (c) continue to hold. Part (a) of Lemma 3 still holds, but parts (b) and (c) do not. A disputant's decision about whether to submit a given case will be unaffected by his own reputation parameter or that of his opponent because the equilibrium level of enforcement will be invariant to ρ. So Lemma 4 fails because a disputant no longer derives benefit from having a reputation as a free-rider. By the derivations in the Proof of Proposition 1, no player will ever be willing to impose punishments, which means that Court rulings will not be enforced in equilibrium. As such, the court will never be used because litigation will amount to a costly lottery over outcomes that are equivalent to bilateral bargaining outcomes.

7.2 Less (but Positive) Enforcement for Free-Riders

Suppose that in order to maintain her reputation as a provider, every non-disputant is expected to provide a positive level of enforcement for all Court rulings—$\hat{e}_n > 0$ always—but less severe punishments are imposed on behalf of free-riders then on behalf of providers.

Suppose that i wins adjudication. Let $c_{ij} = c'$ if i is a free-rider and $c_{ij} = c''$ if i is a provider, where $0 < c' < c''$. Part (a) of Lemma 2 no longer holds, but parts (b) and (c) still hold. Also, part (a) of Lemma 3 still holds. Consider the decision by disputant i about whether to submit a case:

$$\begin{aligned} x^B &< x^I(\rho = (0,0)) = x^I(\rho = (0,1)) \leq x^I(\rho = (1,0)) = x^I(\rho = (1,1)) \\ x^B &> x^J(\rho = (0,0)) = x^J(\rho = (1,0)) > x^J(\rho = (0,1)) = x^J(\rho = (1,1)) \end{aligned}$$

So providers are (weakly) more likely to sue than free-riders, and free-riders are (weakly) more likely to be sued than providers. Direct application of the Proof of Lemma 4 demonstrates that:

$$V_n(\rho = (1,0)|\alpha_m) \geq V_n(\rho = (0,0)|\alpha_m) \quad \text{and} \quad V_n(\rho = (1,1)|\alpha_m) \geq V_n(\rho = (0,1)|\alpha_m)$$

Note that these equations only hold at equality if c' is sufficiently large that bargaining outcomes are invariant to differences between c' and c". So if c' is sufficiently small, these inequalities hold strictly. Note from the Proof of Proposition 1 that if these inequalities hold strictly, then an enforcer who has accepted jurisdiction of the Court is willing to impose costly punishments in order to preserve her reputation, and the upper bound on the costs that she is willing to impose is increasing in her expected benefit from preserving her reputation as a provider of enforcement.

All other results follow directly.

8 Conditioning Enforcement Thresholds on Decision-Nodes

Suppose that each non-disputant's enforcement threshold (i.e. the level of enforcement necessary for the player to preserve her reputation as a provider) is a function of the decision-node. So the enforcement threshold can vary as a function of the identity of the disputants or other elements of the game.

Let $\mathbf{D}_n^e$ denote the set of all decision-nodes at which player n has not been chosen as a disputant, a Court ruling is violated, and n must decide how much enforcement to provide. Then an enforcement strategy is a function $e_n : \mathbf{D}_n^e \rightarrow \mathbf{R}_+$, where $e_n(d)$ denotes the level of enforcement provided by n at a decision-node $d \in \mathbf{D}_n^e$. Similarly, an enforcement threshold is a function $\hat{e}_n : \mathbf{D}_n^e \rightarrow \mathbf{R}_+$, where $\hat{e}_n(d)$ denotes the level of enforcement that player n must provide at a decision-node $d \in \mathbf{D}_n^e$ when the winner is a provider.

This yields the following definition of the reputation variable. For each $n \in N$ and history of actions h^t, let $\rho_n(h^t) = 0$ if there exists a past period t' in which n was a disinterested player, n reached a decision-node $d \in \mathbf{D}_n^e$, the winner of adjudication was a provider, and n chose enforcement $e_n(d) < \hat{e}_n(d)$. Otherwise, let $\rho_n(h^t) = 1$.

Lemma 2 and Propositions 1 and 3 hold when the terms e_n and $\hat{e}_n$ are converted to their conditional forms of $e_n(d)$ and $\hat{e}_n(d)$. All other results continue to hold.

9 Numerical Example in Proof of Proposition 3

The following R code can be used to replicate the numerical example in the published version of the Proof of Proposition 3:

Parameters for the beta distribution:

```
a <- 4
b <- 1
mu <- a/(a+b)
```

Free parameters:

```
k <- 0.000001
e <- 0.499995
```

Simulating the normalizing constant for the density function for a sample of values on the $[0, 1]$ interval:

```
x <- seq(0,1,length=10000)
g <- x^(a-1) * (1-x)^(b-1)
N <- 1/sum(g)
```

Calculating the density function for the subintegral that I care about, $(k/\epsilon, 1)$:

```
low <- k/e
v <- NULL
for(i in 1:length(x)){
    if (low < x[i]) v <- c(v, x[i]) }
f <- N * v^(a-1) * (1-v)^(b-1)
```

Integrand as a function of v:

```
int <- (e*(v-mu)-2*k)*f
```

Integral:

```
sum(int)
```